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A fuzzy approach to multidimensional Material Deprivation measurement: the case of foreigners living in Italy

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Abstract

This paper provides a new approach to the measurement of multidimensional material deprivation, based on partial order theory and on fuzzy set measurement. The main feature of the methodology is that the information needed for the deprivation assessment is extracted directly from the relational structure of the dataset, avoiding any kind of scaling and not proper aggregation procedure, so as to respect the measurement level of the data. An empirical illustration, using data from a special EU-SILC survey on migrants in Italy, provides a new insight on the material deprivation of foreigners living in Italy.

Keywords: partial order theory, fuzzy sets, migration, deprivation

1. Introduction

According to Sen (1985) the notions of "capabilities" and "functionings" should guide the evaluation of individuals' living conditions but unfortunately the informational basis necessary to implement Sen's approach is often not available. On the contrary detailed information on the standard of living of the households, in terms of income and/or durable goods affordable to them, is more easily available through most of the household sample surveys. In particular, such latter kind of information is also more reliable than income data, because it allows to overcome difficulties such as unrecorded income sources, illegal transactions, or simply underreporting. These reliability issues raise especially dealing with foreign populations, for which the study of standard of living is particularly complex.
It is anyway true that the knowledge of the types and amount of goods with which various individuals are endowed do not necessarily allow us to draw conclusions on their standard of living or quality of life (Cohen, 1993). Moreover, the use of information on the ownership of durable goods let emerge the issue of defining and measuring multidimensionally the standard of living of individuals. Indeed taking a multidimensional point of view requires therefore to have available new measures of the standard of living and, in recent years, several and various techniques have been proposed to estimate the standards of living and quality of life.

In this paper we chose to focus on the material deprivation as a complementary piece of information in the standard of living of foreigners in Italy. A new approach to the analysis of multidimensional material deprivation data is adopted. This is accomplished by means of partial order theory, a set of algebraic tools that provides the right formal language to tackle evaluation problems. The main feature of the approach is that the measurement level of the data is fully respected, avoiding any kind of not proper scaling.
For the sake of simplicity the necessary algebraic tools will be presented very briefly and the application on material deprivation of foreigners living in Italy for the year 2009 is only rough out.

2. Measuring multiple deprivation

In an effective review provided by Deutsch and Silber (2005) four main strands in the measurement of multidimensional poverty are insulated: the distance function approach, the information theory approach, the axiomatic approach to indices, and the fuzzy approach. They can be easily extended to the measurement of multidimensional material deprivation. We focus here on the last two, due to some connections to our approach in this paper.

Few studies have attempted to derive axiomatically multidimensional indices of poverty, among these Chakravarty *et al*. (1998), maybe the first published article that follows this approach,[1] and more recently Tsui (2002), who follows his earlier work on axiomatic derivations of multidimensional inequality indices (see Tsui, 1995, 1999). The basic idea behind Chakravarty *et al*. (1998), as well as Tsui's (2002) approach, is that an index of multidimensional poverty is an aggregation of shortfalls of all the individuals, where the shortfall with respect to a given need reflects the fact that the individual does not have even the minimum level of that basic need.

The *Fuzzy Set Approach* was originally developed by Zadeh (1965) on the basis of the idea that certain kinds of objects may not be sharply attributed to a single class. He maintained that sometimes one is unable to determine which elements belong to a given set and which ones do not. Zadeh himself (1965) characterized a fuzzy set (class) as "*a class with a continuum of grades of membership*". Along last decades, this notion was naturally extended to the concept of poverty, which is an attribute with several nuances other than certainly poor and certainly not poor (see Cerioli and Zani, 1990, Cheli *et al.*, 1994, Cheli and Lemmi, 1995). The fuzzy nature of the membership of an individual to the set of poor people (or to that of the deprived ones) is particularly evident when one takes a multidimensional approach to poverty measurement. This happens when according to some dimensions one would certainly define people as poor whereas according to others as not poor.

This paper proposes new tools for fuzzy analysis of multidimensional material deprivation data and, more generally, for the analysis of multivariate ordinal datasets for evaluation and ranking purposes.

3. Data and descriptive statistics on the material deprivation among migrants in Italy

In all European countries the migrant population tends to have worst living conditions: higher at risk of poverty, very low work intensity, severe material deprivation and housing deprivation and segregation (Guio 2005 and 2009; Lelkes and Zólyomi, 2011; Eurostat, 2011). The situation of foreigners living in Italy follows this general rule. According to Istat (2011) in Italy more than one out of three families with foreigners (34.5%) lives a situation of material deprivation -according to the Eurostat definition- against 13.9% of families with only Italian members. This deprivation gap is more relevant in Northern and Central regions than in the Southern ones. Just the intensity of material deprivation is stronger among families with foreigners: 53.4% of deprived families is strongly deprived against 43.2% among Italian deprived families.

Our empirical analyses are made on a special EU-SILC survey on "Income and living conditions" of foreigners and intends to show the diffusion and the intensity of material deprivation among people of different nationalities. The survey was carried out by Istat in 2009 on a sample of 6,000 households resident in Italy with at least one foreign member. It has the same methodological tools used for the standard EU-SILC survey (questionnaires, survey techniques, methods of correction, imputation and integration of data, *etc*.) and it allows comparisons with the deprivation of the households in the standard EU-SILC Italian sample.

[1] According to Chakravarty *et al*. (1998) the axioms at the basis of a multidimensional poverty measure are: Symmetry, Focus, Monotonicity, Principle of Population, Continuity, Non-Poverty Growth, Non-decreasingness in Subsistence Levels of Basic Needs, Scale Invariance, Normalization, Subgroup Decomposability, Factor Decomposability, Transfer Axiom, and Non decreasing Poverty under Correlation Increasing Arrangement.

The preliminary descriptive analysis of material deprivation of foreigners reveals strong and interesting differences among nationalities. These general evidences conceal an extremely heterogeneous situation that change a lot among nationalities. Indeed foreigners in Italy belongs to a wide variety of nationalities (almost 200) with the first 10 that represent only 63.8% of the overall foreign population. In particular at the end of 2010 the largest foreign group were Rumania (21.2%), Albania (10.6%), Morocco (9.9%), China (4.6%), and Ukraine 4.4%.

As a general pattern, we can outline that migrants from outside the European Union are occasionally more exposed to disadvantages than the natives population. EU and non-EU migrants constitute two rather distinct groups in most countries in terms of their exposure to detrimental outcome. The disadvantage of non-EU migrants tends to be large also in relative terms: in all EU countries the difference between the local population and non-EU migrants is wider, than to the EU migrants. In particular if we look at diffusion and intensity of material deprivation it is evident that they vary a lot between EU and non-EU migrants. The incidence reaches 30% for EU migrants and 15% for non-EU migrants and, the intensity is respectively of 2.4 and 2.9.

4. Measures and methods

As briefly mentioned before, the main feature of the approach here adopted is that the measurement nature of the data is fully respected, avoiding any kind of scaling and aggregation procedure. This is accomplished by means of partial order theory, a set of algebraic tools that provides the right formal language to tackle ordinal evaluation of individuals' material deprivation.

The basic idea is quite simple. Let $v_1,...,v_k$ be k dichotomous deprivation dimensions ($i = 1,...,k$). To each individual in the population, a profile $\boldsymbol{p} = (p_1,..., p_k)$ is associated, whose components are the statuses of the statistical unit on the deprivation dimensions (either 0 or 1). The set P of profiles is turned into a partially ordered set (or *poset*) defining $\boldsymbol{p} < \boldsymbol{q} \Leftrightarrow p_i \leq q_i \ \forall\, i = 1,...,k$ where $\boldsymbol{q}$ is a generic deprivation profile of any other individual in the population. The set of relationships among profiles can be graphically represented by the Hasse diagram (Davey and Priestley, 2002). In this framework, a multidimensional deprivation threshold T is a minimal set of profiles such that any profile below one of its elements is classified as deprived. Given the threshold, any other profile may be assessed in terms of deprivation, based on its position with respect to T, in the "profile poset". Technically, this is achieved representing the partial order as the intersection of all of its linear extensions, and counting over them as described in Fattore *et al.* (2011). The resulting evaluation function assigns to each profile (and thus to any individual sharing it) a score in [0,1], representing the degree of deprivation given T. In practice, the procedure quantifies the degree of ambiguity in the classification of a profile into the set of deprived profiles and may be better interpreted as a way to compute a fuzzy membership function. What is relevant here, is that the focus is on profiles and information is extracted out of the mathematical structure representing the basic relation existing among them, i.e. the partial order relation.

Given the membership function, one can compute fuzzy extensions of classical material deprivation indicators. Particularly, one can compute the fuzzy analogue of the Head Count Ratio, simply averaging over the membership degrees of the statistical units in the population, and the fuzzy intensity of material deprivation, thus providing a complete set of synthetic indicators for its assessment.

In our paper we refer to the classical nine items of deprivation used for the official measure by Eurostat (2012): inability to 1) face unexpected expenses; 2) have one week annual holiday away from home; 3) pay for arrears; 4) have a meal with meat, chicken or fish every second day; and 5) keep home adequately warm; and enforced lack of 6) a washing machine; 7) a color TV, 8) a telephone, 9) a personal car. All these items are dichotomous and we will provide a different way to order foreigners living in Italy on the basis of their material deprivation. The results of the outlined approach are compared to those achieved through more classical procedures.

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