Divergence/convergence in life-spans across developed countries in 1970-2010 based on the concept of the Equivalent Length of Life

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FIRST DRAFT

Abstract

Based on the concept of the Equivalent Length of Life (ELL) as proposed by Silber (1983), we quantify differences in the distribution of ages at death across developed countries and study convergence/divergence of their mortality distributions. Advantage of the EEL over previously used indicators lies in the fact that it allows to compare distribution of ages of death taking into account up to three parameters of the distribution at the same time, that is life expectancy, dispersion and asymmetry of ages at death. Total inequality between countries is further decomposed into a contribution of selected parameters of the distribution and its changes over calendar time. As an inequality measure we apply either standard deviation or Gini index. We study differences in age-at-death distributions and their convergence/divergence across the countries of the Human Mortality Database in 1970-2010.

Our results show diverging pattern of mortality in developed countries over the years 1970-2000 for both sexes. Since the mid-2000, however, we observe a convergence in the mortality distributions. The largest contributing factor to the total inequality, as well as changes in the indices over the calendar time, are differences between countries in the mean age at death. A large contribution to the total inequality and its changes over the calendar time was that of a negative and growing covariance between the mean and standard deviation of ages at death. Similar,

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an important and growing contribution was that of covariance betwen the mean and a negative skewness of the distribution of ages at death.

Introduction

Studies on determinants of mortality or social development rely on methods to quantify differences in mortality between regions or populations. As genetic factors play a small role in differences in age at death (according to the studies based on Danish twins by McGue et al. (1993) or Herskind et al. (1996)), variation in mortality to a large extent results from an unequal distribution of resources, other ecological factors or behavioural differences. This line of argument has been already discussed in detail in the large body of demographic literature on socio-economic differences in mortality within populations (for example, European studies by Mackenbach et al. (1997), Mackenbach et al. (2003), or Kunst et al. (2004)). Differences in age at death are often used as an ultimate measure of unequal distribution of resources between populations (Asada, 2006).

To a great extent a universal measure in the demographic literature to quantify population health and compare across countries (or populations) is life-expectancy at birth. It is the most common indicator of changes in survival over calendar time (Oeppen and Vaupel, 2002; White, 2002, for example), and forms the basis for the discussion on differences in population health across countries (Goesling and Firebaugh, 2004; Moser et al., 2005, for example) or to compare mortality across other sub-groups of population (for an review of studies that discuss health differences across socio-economic groups based on inequalities in life-expectancies are, for example, Mackenbach et al. (1997), Mackenbach et al. (2003) or Kunst et al. (2004)). While life expectancy at birth is a useful summary of mortality across all age groups, it refers to only one measure of the mortality distributions, that is the mean age at death in a stationary population. There is a growing agreement in the demographic literature that differences in population health across various groups should be discussed based also on additional to life expectancy measures of mortality distributions. For example, Normal duration of life, that is the modal age at death, was studied already by Lexis (1878) and recently by Cheung et al. (2005), Canudas-Romo (2010) and Thatcher et al. (2010). Furthermore, unequal distribution of ages at death was discussed in the framework of *compression-rectangularization* hypothesis (Engelman et al., 2010; Fries, 1980; Shkolnikov et al., 2003; Vaupel et al., 2011; Wilmoth and Horiuchi, 1999, for example).

To our knowledge three studies so far have studied convergence/divergence of distributions of ages at death across a group of countries applying measures different from life-expectancy. Although the research question of the study is different from our main focus, the study of Smits and Monden (2009) is worth mentioning here. The authors calculated Theil and Gini indices over the distribution of age at death to study length of life inequality in the World. Theil index was further decomposed to within- and between-country inequalities. Similar, Edwards (2011) studied global inequality in the length of life based on standard deviation, interquartile range, Gini coefficient and Theil index and discussed results of a decomposition of the variance and the Theil index into within- and between-country contribution to inequality in life durations. The study of Edwards and Tuljapurkar (2005) applied the Kullback-Leibler measure of divergence to quantify similarities in mortality distributions between countries and study their developments over calendar time. Limitation of the study by Edwards and Tuljapurkar (2005) comes from the fact that the baseline distribution for comparisons was that of Sweden in 2002 and given that distributions of ages at death of selected countries become more similar to the Swedish distribution it does not necessarily guarantee convergence of the age-at-death distributions between those countries. In addition, in the study by Edwards and Tuljapurkar (2005) distributions are compared pairwise and the convergence between the countries is assessed based only on a visual inspection of the statistics. Hence, fixing the problem of a benchmark distribution for comparisons, would only allow a visual comparison to study the trends.

The aim of this study is to propose a set of measures to quantify differences in distribution of ages at death across a group of countries to enable a formal study of convergence/divergence of the distributions over the calendar time. Further, the total inequality between countries is decomposed into the contribution of mean, inequality and asymmetry of the distribution of ages at death to the total inequalities. Based on the proposed indicators, we would discuss convergence/divergence in mortality in selected groups of countries and decompose total change in inequality into change in differences between the following parameters of age-at-death distributions: life-expectancy, inequality and skewness.

Contribution of inequality of ages at death to total inequality in our study is different from the contribution of within-countries inequalities in the study of Smits and Monden (2009) and Edwards (2011). Since within-country component of total variation is a sum of variations within single countries it does not serve our purpose to compare mortality distributions between countries. Our study also overcomes mentioned above limitations to that of Edwards and Tuljapurkar (2005).

In this study, mortality distributions in single countries are summarized by index of Equivalent Length of Life (ELL) as proposed by Silber (1983). Inequality between the values of the ELL in a group of countries would be quantified in selected calendar years by standard deviation and a Gini Index.

Data and Methods

Data used in the study comes from the Human Mortality Database (2011). Out of the 38 counties present in the database, Chile, Israel and Slovenia were excluded due to a short time period covered. Fo the remaining countries, we estimated the statistics for every five years in the common period covered: between 1970 and 2010 (or the last year available but not earlier than 2008).

Similar to the previous studies of inequality before death (Edwards, 2011; Edwards and Tuljapurkar, 2005; Smits and Monden, 2009), we eliminate differences in infant and childhood mortality, studying truncated distributions of length of life above age of completed 10 years.

From life-tables of single countries we estimated a value of the Eqivalent Length of Life for those aged 10. The concept of the *Equivalent Length of Life* (ELL) was introduced Silber (1983) as a development indicator, based on the concept of Atkinson (1970) and Kolm (1976a,b) to quantify inequality, and it measures "…length of life which, if being identical for all individuals, would give the same social welfare as the actual distribution of deaths by age." (p.21). Selecting the formula for *ELL* one has to decide on a scale invariance, that it a response of the statistic to proportional and absolute equal change in life-time durations of individuals. Both choices are then incorporated into the formula following the Social Welfare Function concept:

$$ELL = e_{10}(1 - I)$$
 (1)

where *I* stands for any inequality index.

We apply two groups of measures according to the type of scale invariance. The first set of mea-

sures is invariant to absolute equal change, while the second set is invariant to equal relative change in life-time durations of individuals.

According to Silber (1983), one more choice is to be made concerning the statistic under study. One should decide whether to give equal weight to differences in length of life at all ages or the weights are different, for example, for younger age-groups. As our main research question concerns differences in distributions of ages at death and we do not intent to place any moral judgment on that differences, we give equal weight to inequalities in mortality at all ages under study.

Measures based on statistics invariant to equal absolute change

First inequality measure employed in this study is invariant to equal absolute changes in life duration of all individuals. In this case we employ coefficient of variation as an inequality measure which reduces ELL to:

$$e_C = e_{10}(1 - \frac{SD}{e_{10}}) = e_{10} - SD \tag{2}$$

where *SD* stands for a standard deviation of age at death for ages 10 and older in a life-table.

In this part of the study, inequalities in the value ELL across the developed countries are quantified with variance and total variance of the distribution of ELL is decomposed into the variance of the two components of ELL and their covariance according to the formula that is derived from a simple decomposition of variance of the sum of two variables:

$$S^{2}(e_{C}) = S^{2}(e_{10}) + S^{2}(SD) - 2cov(e_{10}, SD)$$
(3)

The first term stands for inequalities in life-expectancies as measured by variation in e_{10} and the second term stands for inequalities in dispersion parameters as measured by variation in *SD*. The third term indicates covariance between e_{10} and *SD* across the group of countries under study.

Similar, when studying changes in inequalities in *ELL* in a group of countries, the contribution

of shifts in life-expectancy and in inequality of ages at death into changes in the total inequality in ELL across the countries can be assessed accordingly.

Furthermore we take into account asymmetry of the distribution of ages at death in a modified Equivalent Length of Life, e_A that is defined in this case as:

$$e_A = e_{10} - SD(1 - A) \tag{4}$$

where A is a meassure of asymmetry. In this case it takes form

$$A = \frac{1}{2} \frac{SD_U - SD_L}{SD} \tag{5}$$

where SD is standard deviation of ages at death of the whole distribution above age of completed 10 years, SD_U is standard deviation of ages at death of those who live longer than the median age at death and SD_L stands for Standard Deviation in ages at death of those who live less years than the median age at death and completed 10 years of age. Median age at death is estimated for the distribution of ages at death from truncated distributions of length of life above age of completed 10 years.

As a result e_A appears as

$$e_A = e_{10} - SD + \frac{1}{2}(SD_U - SD_L) = e_C + \frac{1}{2}(SD_U - SD_L)$$
(6)

Inequalities in the value of e_A across the developed countries are quantified with variance. Next, total variance of e_A is decomposed according to a simple decomposition of variance of the sum of two variables:

$$S^{2}(e_{A}) = S^{2}(e_{10}) + S^{2}(SD) - 2cov(e_{10}, SD) + S^{2}(A') - 2cov(e_{10} - SD, A') = S^{2}(e_{C}) + S^{2}(A') + 2cov(e_{C}, A')$$
(7)

where $A' = \frac{1}{2}(SD_U - SD_L)$.

The first element stands for inequalities in life-expectancies as measured by variation in e_{10} , the second element stands for inequalities in dispersion parameters as measured by variation

in *SD*, and the third element indicates covariance between e_{10} and *SD* across the group of countries under study, the last two elements stand for differences in the asymmetry parameter of the distributions and covariance between ELL and the asymmetry parameter: $(cov(e_{10} - SD, A') = cov(e_C, A'))$.

Measures based on statistics invariant to equal relative change

As demonstrated by Silber (1992), when Gini's Concentration Ratio of ages at death is applied as an inequality measure, ELL reduces to:

$$e_C = e_{10} - \frac{1}{2}\Delta \tag{8}$$

where Δ is Gini's Mean Difference in ages at death defined as:

$$\Delta = \frac{1}{N^2} \sum_{i=10}^{l} \sum_{j=10}^{l} d_i d_j |i-j|$$
(9)

and

$$N = \sum_{i=10}^{l} d_i \tag{10}$$

Inequalities in the value of ELL across the developed countries are quantified separately in each calendar year with Gini index and, according to the decomposition formula proposed by Kakwani (1977), the total inequality is decomposed into the inequality between life-expectancies and inequality in the dispersion parameters of the single distributions of ages at death. Hence, total value of Gini Index of the ELL in countries under study is decomposed into:

$$G(e_C) = \frac{\overline{e_{10}}}{\overline{e_C}} c(e_{10}) - \frac{1}{2} \frac{\overline{\Delta}}{\overline{e_C}} c(-\Delta)$$
(11)

where c(.) stands for Kakwani concentration coefficient which may be estimated as follows (Lerman and Yitzhaki, 1984):

$$c(e_{10}) = 2cov(\frac{e_{10}}{e_{10}}, (1 - F(e_C)))$$
(12)

and

$$c(-\Delta) = 2cov(\frac{\Delta}{\Delta}, (1 - F(e_C)))$$
(13)

, where $F(e_C)$ stands for cumulative distribution of Δ across countries.

Hence, the first term stands for contribution of inequalities in life-expectancies to the total inequality in ELL, while the second one stands for inequalities in the dispersion parameters.

Furthermore, Silber (1988) takes into account asymmetry of the distribution of ages at death in a modified Equivalent Length of Life, e_A that is defined as:

$$e_A = e_{10} - \frac{1}{2}\Delta(1 - A) \tag{14}$$

where, the measure of asymmetry is defined by (9)

$$A = \frac{1}{2} \frac{\Delta_U - \Delta_L}{\Delta} \tag{15}$$

and Δ stands for the mean inequality of ages at death of the whole distribution of ages at death (above age of completed 10 years), Δ_U is the mean inequality of ages at the death of those who live longer than the median age at death and Δ_L stands for the mean difference in ages at death of those who live less years than the median age at death and at least 10 years. Median age at death is estimated for the distribution of ages at death from truncated distributions of length of life above age of completed 10 years.

As a result e_A can be reduced to

$$e_A = e_{10} - \frac{1}{2}\Delta + \frac{1}{4}(\Delta_L - \Delta_U)$$
(16)

Therefore when inequalities in the value of modified ELL across countries are quantified separately in each calendar year with Gini index, the total inequality is decomposable to the inequality between life-expectancies, inequality in the dispersion of ages at death and inequality in the asymmetry of the distribution. In this case, asymmetry of the distribution is quantified as difference between the mean inequality in ages at death of those who live more years than the median and the mean inequality in ages at death of those who live less years than the median:

$$A' = \frac{1}{4} (\Delta_L - \Delta_U) \tag{17}$$

Hence total value of Gini Index of the e_A in countries under study is can be decomposed into:

$$G(e_A) = \frac{\overline{e_{10}}}{\overline{e_A}}c(e_{10}) - \frac{1}{2}\frac{\overline{\Delta}}{\overline{e_A}}c(\Delta) + \frac{\overline{A'}}{\overline{e_A}}c(A')$$
(18)

This part of the calculations was done with packages *ineq* and *IC2* in R.

Results

Table 1 presents mean values of the life-table summary measures by sex that are either based on variance, which is a statistic invariant to absolute change, or on Gini's Mean Difference in ages at death, which is invariant to equal relative change.

Over the years 1970-2010, the mean value of expected number of years lived at age 10 (e_{10}) in developed countries increased with every next decade for both sexes. At the same time, the mean value of the *ELL*, whether life-expectancy was adjusted only for inequality of ages at death (e_c) or for both inequality and skewness of ages at death (e_a), also increased. This increase was present in both type of measures: for e_{10} adjusted for inequality statistics invariant to equal absolute change, as well as, statistics invariant to equal relative change. The opposite development was observed for the mean value of inequality of life-durations of individuals, in both type of statistics: Standard Deviation (*SD*) and Gini's Mean Difference (Δ). At the same time the distributions of ages at death became more negatively skewed for both sexes (*A* in both types of measures), which resulted for both sexes from larger decrease in dispersion of ages at death below the median age at death as compared to drop in dispersion of ages at death above the median age (results not shown in Tables).

Variation in the life-table summary measures in the years 1970-2010 are shown on Figure 1. On Figure 2 we present values of the Gini Index for the life-table summary measures. We also quote values of the statistics every ten years over the study period, together with a contribution of parameters of mortality distributions to the total inequality, in Tables 2 and 3, accordingly. Tables 4 and 5 report changes in the statistics under study every ten years and over the overall study period. In the years 1970-2000, for both sexes inequality in life-time durations in the studied group of countries was the largest for the indeces based on the *ELL* that is expected number of years lived at age ten adjusted for additional parameters of the distribution of ages at death: variation and/or skewness. Which of the values of *ELL* was larger varied between sexes, measures of inequality applied, as well as, over calendar time. In the overall study period, the reported above growing mean values of the statistics under study, coexisted with a divergence in mortality distributions. The pattern of changes in the indices of inequality over the calendar time were independent of the type of inequality measure applied. For both sexes, in total the largest change between 1970 and 2010 characterized expected number of years live adjusted for both inequality and skewness of the distribution and the smallest change was that of the unadjusted measure. As far as development over the calendar time is concerned, three subperiods can be distniguished: (1)years between 1970 and early 1990s, characterized by a steady divergence between countries; (2) short period of convergence in the distributions in the 1990s, followed by a divergence until mid-2000; (3) convergence in the distributions since the mid-2000.

In the study years, variance in the mean ages at death (e_{10}) was the largest contributor to the variance in *ELL* (Table 2) and also divergence between countries in mortality distributions resulted for both sexes mainly from an increase in inequalities in the mean values of ages at death. This result is reflected in the contribution of changes in e_{10} to shifts in e_C and e_A in Table 4. Similar, for both sexes, inequality in mean ages at death was the largest contributing factor to differences in the distributions of ages at death quantified by the Gini Index (Table 3) and to growing inequalities as reflected in change in the values of the indeces in Table 5.

The second largest contribution to the total variance in e_C and e_A for both sexes was that of covariance between e_{10} and SD. Contribution of covariance between e_{10} and SD to total inequality in ELL also grew over study time. The covariance between e_{10} and SD was itself negative, which indicates that countries with higher level of life-expectancy, where also those characterized by lower inequality of ages at death. This result confirms that of Vaupel et al. (2011), who reported that populations with high life-expectancy are also among those with low disparity of ages at death. On the other hand, the increasing contribution of covariance between e_{10} and SD stands for a growing negative covariance between the variables for two sexes and is a result of opposite developments of SD and e_{10} over the calendar time in the group of countries (results not shown in Tables).

The contribution of differences in the skewness statistic (A') to the total inequality in e_A was small for both sexes and for both type of indeces of inequality. However, a large effect of asym-

metry on variance in *ELL* was present for males in the contribution of negative covariance between e_C and A' and this effect grew over calendar time. Negative covariance between e_C and A', together with a negative skewness, indicates that countries with higher level of *ELL* are also those with higher level of negative assymetry of the distribution.

The growing contribution of covariance between e_C and A' for both sexes stands for a growing negative covariance between the variables and is a result of opposite development in e_C and A'over the calendar time, which means that large increase in e_C was associated with the highest increase in the negative skewness of the distribution of ages at death in the studied group of countries (results not shown in Tables). For women, relationship between changes in e_{10} and the skewness parameter dependent on the study sub-period: change in the covariance in the first two decades was negative and later on – positive.

Altogether, for males, according to the statistics invariant to equal absolute change, increase in the mean age at death (e_{10}) in a single country was negatively related to change in the inequality of ages at death and positively with the level of negative skewness of the distribution. For women, the relationship between changes in the mean age at death and in variation in ages at death was similar to that of men, but the relationship between e_{10} and the skewness parameter dependent on the period.

1 Summary

We propose a set of measures to quantify differences in distribution of ages at death across a group of countries to study convergence/divergence of the distributions of ages at death in developed countries from the Human Mortality Database (2011) over the years 1970-2000. The proposed statistics are based on the Index of Equivalent Length of Life (ELL) of Silber (1983). We apply measures invariant to absolute or relative equal change in mortality distributions. The total inequality between countries is further decomposed into the contribution of mean, inequalities and asymmetry of the distribution of ages at death.

Over the years 1970-2000, for both sexes we observe an increase in inequality in life-time durations in the studied group of countries. The largest increase characterized the value of *ELL* with expected number of years lived at age ten adjusted for variation and skewness of the distribution. The pattern of changes in the indices of inequalities over the calendar time were independent of the type of inequality measure applied and three periods with distinct developments were distinguished: (1)years between 1970 and early 1990s, characterized by a steady

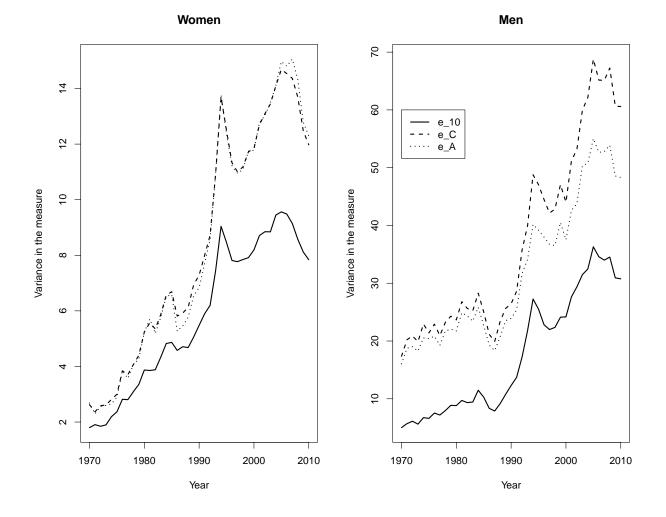


Figure 1: Variance in the life-table summary measures in developed countries, 1970-2010

Source: Authors' estimations based on Human Mortality Database (2011)

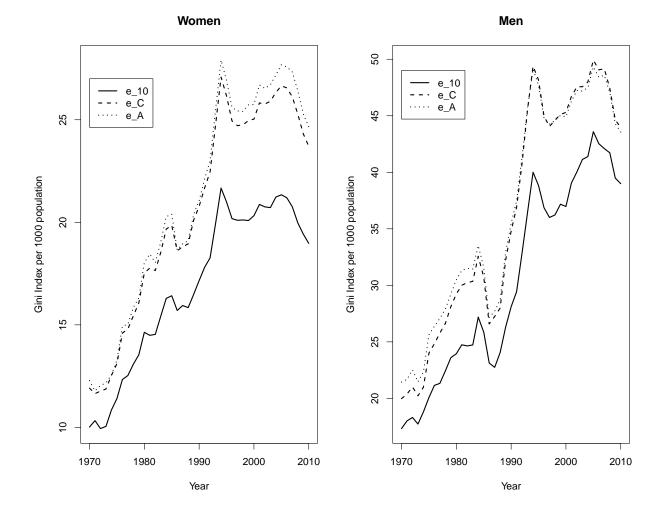


Figure 2: Gini Index of the life-table summary measures in developed countries, 1970-2010

Source: Authors' estimations based on Human Mortality Database (2011)

divergence between countries; (2) convergence in the distributions in the mid-1990s, followed by a divergence until mid-2000; (3) convergence in the distributions since the mid-2000. Inequality in the mean ages at death (e_{10}) was the largest contributor to differences in *ELL* between developed countries. It was also the largest contributing factor for divergence between countries in mortality distributions resulted for both sexes. An important contributor to the variance in e_C and e_A was covariance between e_{10} and *SD*, as well as, for males covariance between e_C and A'. In addition, contribution of the these two measures to *ELL* grew over the study period. Altogether, negative covariance between e_{10} and *SD* (or A') indicates that countries with higher level of life-expectancy, where also those characterized by lower inequality (higher negative skewness) of ages at death.

Statistic	1970	1980	1990	2000	2010*				
	Women								
<i>e</i> ₁₀	75.58	76.84	78.12	79.52	81.48				
Statistic	Statistics based on measures invariant to equal absolute change								
e_C	62.24	63.66	65.04	66.63	68.86				
e_A	58.71	60.15	61.47	62.97	65.16				
SD	13.34	13.18	13.08	12.90	12.62				
Α	-0.26	-0.27	-0.27	-0.28	-0.29				
Statistics	Statistics based on on measures invariant to equal relative change								
e _C	68.52	69.87	71.21	72.74	74.87				
e_A	66.86	68.21	69.53	71.01	73.11				
Δ	14.13	13.95	13.81	13.57	13.21				
A	-0.23	-0.24	-0.24	-0.26	-0.27				
	Men								
<i>e</i> ₁₀	69.24	69.74	70.91	72.44	74.74				
Statistic	s based	on mea	sures in	variant to	equal absolute change				
e_C	53.91	54.49	55.67	57.39	60.07				
e_A	50.48	51.11	52.24	53.94	56.73				
SD	15.32	15.25	15.24	15.04	14.68				
A	-0.23	-0.23	-0.23	-0.24	-0.24				
Statistics based on measures invariant to equal relative change									
e _C	61.38	61.91	63.11	64.79	67.58				
e_A	59.70	60.24	61.43	63.10	65.90				
Δ	16.17	16.12	16.09	15.85	15.36				
Α	-0.20	-0.20	-0.21	-0.21	-0.22				
	•		*or latest a	vailable vear					

Table 1: Mean values of the life-table summary measures based on statistics invariant to absolute and relative change in mortality in developed countries, 1970-2010

*or latest available year

Statistic	1970	1980	1990	2000	2010*		
Women							
$S^2(e_{10})$	1.80	3.87	5.48	8.19	7.84		
$S^2(e_C)$	2.62	5.24	7.28	11.84	11.97		
$S^2(e_A)$	2.68	5.24	6.84	11.78	12.30		
Contril	oution to	o the va	riance ii	$h e_C of$			
$S^2(e_{10})$	1.80	3.87	5.48	8.19	7.84		
$S^2(SD)$	0.45	0.44	0.32	0.52	0.65		
$-2cov(e_{10},SD)$	0.37	0.93	1.49	3.13	3.49		
Contril	oution t	o the va	riance ii	$h e_A of$			
$S^2(e_{10})$	1.80	3.87	5.48	8.19	7.84		
$S^2(SD)$	0.45	0.44	0.32	0.52	0.65		
$cov(e_{10},SD)$	0.37	0.93	1.49	3.13	3.49		
$S^2(A')$	0.05	0.07	0.06	0.06	0.06		
$2cov(A', e_C)$	0.01	-0.07	-0.51	-0.12	0.26		
Men							
$S^2(e_{10})$	5.01	8.82	12.31	24.17	30.76		
$S^2(e_C)$	17.37	23.65	26.45	44.02	60.58		
$S^2(e_A)$	16.06	21.74	23.93	37.53	48.32		
Contribution to the variance in e_C of							
$S^2(e_{10})$	5.01	8.82	12.31	24.17	30.76		
$S^2(SD)$	4.97	5.45	4.94	5.48	7.10		
$-2cov(e_{10},SD)$	7.38	9.38	9.21	14.36	22.72		
Contribution to the variance in e_A of							
$S^2(e_{10})$	5.01	8.82	12.31	24.17	30.76		
$S^2(SD)$	4.97	5.45	4.94	5.48	7.10		
$-2cov(e_{10},SD)$	7.38	9.38	9.21	14.36	22.72		
$S^2(A')$	0.16	0.15	0.25	0.46	0.86		
$2cov(A', e_C)$	-1.48	-2.06	-2.77	-6.94	-13.13		

Table 2: Variance in the life-table summary measures based on statistics invariant to absolute change in developed countries, 1970-2010. Contribution of selected parameters of mortality distributions to the variance in the summary measures

scriptsize*or latest available year

Source: Authors' estimations based on Human Mortality Database (2011)

Table 3: Inequality in the life-table summary measures based on statistics invariant to relative change in developed countries (per 1000 population), 1970-2010. Contribution of selected parameters of mortality distributions to inequality in the summary measures (per 1000 population)

Statistic	1970	1980	1990	2000	2010*		
Women							
$G(e_{10})$	10.0	14.6	17.2	20.3	19.0		
$G(e_C)$	11.9	17.5	20.8	25.0	23.7		
$G(e_A)$	12.3	18.0	21.1	25.7	24.7		
Contrik	oution t	o $G(e_C)$) of				
e_{10}	10.6	15.9	18.7	22.1	20.6		
Δ	1.3	1.6	2.1	2.9	3.1		
Contrib	oution t	to $G(e_A)$) of				
e_{10}	10.7	16.2	19.1	22.7	21.0		
Δ	1.5	1.7	2.1	3.0	3.2		
Α'	0.1	0.1	-0.2	0.1	0.4		
Men							
e_{10}	17.3	23.9	28.1	37.0	39.0		
$G(e_C)$	22.8	31.5	36.3	47.4	49.3		
$G(e_A)$	24.2	32.7	37.0	47.1	49.2		
Contribution to $G(e_C)$ of							
Total contribution of e_{10}	19.3	26.8	31.5	41.4	43.4		
Total contribution of Δ	3.6	4.6	4.8	6.0	5.9		
Contribution to $G(e_A)$ of							
e_C	19.6	27.5	32.2	42.5	44.4		
Δ	3.8	4.8	5.0	6.2	6.1		
A'	0.8	0.4	-0.2	-1.5	-1.4		

scriptsize*or latest available year

Table 4: Changes in the inequalities in the life-table summary measures based on statistics invariant to absolute change in developed countries. Total change in the inequalities between 1970 and 2010

Statistic1970-19801980-19901990-20002000-20101970-2010*Women $S^2(e_{10})$ 2.071.612.71-0.356.04 $S^2(e_C)$ 2.612.054.560.139.35 $S^2(e_A)$ 2.561.604.940.529.62Change in the contribution to the variance in e_C of $S^2(e_1)$ 2.071.612.71-0.356.04 $S^2(e_1)$ 2.071.612.71-0.356.04 $S^2(e_1)$ 2.071.612.71-0.356.04 $S^2(e_D)$ 0.02-0.120.200.120.19 $cov(e_{10},SD)$ 0.560.561.640.363.12Change in the contribution to the variance in e_A of $S^2(e_C)$ 2.612.054.560.139.35 $S^2(e_C)$ 0.02-0.01-0.000.000.01 $cov(A', e_C)$ -0.08-0.440.380.380.25Men $S^2(e_L)$ 3.813.4911.866.5925.75 $S^2(e_A)$ 5.692.1813.6110.7932.26Change in the contribution to the variance in e_C of $S^2(e_L)$ 3.813.4911.866.5925.75 $S^2(e_L)$ 3.813.4911.866.5925.75 $S^2(sD)$ 0.48-0.510.541.622.13 $cov(e_{10},SD)$ 2.00-0.185.168.35 <th></th> <th></th> <th></th> <th></th> <th></th> <th></th>										
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	Statistic	1970-1980	1980-1990	1990-2000	2000-2010	1970-2010*				
$\begin{array}{c c c c c c c c c c c c c c c c c c c $	Women									
$\begin{tabular}{ c c c c c c c c c c c c c c c c c c c$	$S^2(e_{10})$	2.07	1.61	2.71	-0.35	6.04				
$\begin{tabular}{ c c c c c c c c c c c c c c c c c c c$	$S^2(e_C)$	2.61	2.05	4.56	0.13	9.35				
$\begin{array}{c c c c c c c c c c c c c c c c c c c $	$S^2(e_A)$	2.56	1.60	4.94	0.52	9.62				
$\begin{array}{c c c c c c c c c c c c c c c c c c c $	(Change in th	e contributio	n to the varia	ance in e_C of					
$\begin{array}{c cccc} \hline cov(e_{10},SD) & 0.56 & 0.56 & 1.64 & 0.36 & 3.12 \\ \hline Change in the contribution to the variance in e_A of S^2(e_C) & 2.61 & 2.05 & 4.56 & 0.13 & 9.35 \\ S^2(A') & 0.02 & -0.01 & -0.00 & 0.00 & 0.01 \\ \hline cov(A',e_C) & -0.08 & -0.44 & 0.38 & 0.38 & 0.25 \\ \hline \hline \\ \hline \\ \hline \\ S^2(e_{10}) & 3.81 & 3.49 & 11.86 & 6.59 & 25.75 \\ S^2(e_C) & 6.29 & 2.80 & 17.56 & 16.57 & 43.21 \\ S^2(e_A) & 5.69 & 2.18 & 13.61 & 10.79 & 32.26 \\ \hline \\ \hline \\ Change in the contribution to the variance in e_C of S^2(e_{10}) & 3.81 & 3.49 & 11.86 & 6.59 & 25.75 \\ S^2(e_A) & 5.69 & 2.18 & 13.61 & 10.79 & 32.26 \\ \hline \\ \hline \\ Change in the contribution to the variance in e_C of S^2(e_{10}) & 3.81 & 3.49 & 11.86 & 6.59 & 25.75 \\ S^2(SD) & 0.48 & -0.51 & 0.54 & 1.62 & 2.13 \\ \hline \\ \hline \\ \hline \\ Change in the contribution to the variance in e_A of S^2(e_C) & 6.29 & 2.80 & 17.56 & 16.57 & 43.21 \\ \hline \\ S^2(e_C) & 6.29 & 2.80 & 17.56 & 16.57 & 43.21 \\ S^2(A') & -0.01 & 0.10 & 0.21 & 0.41 & 0.70 \\ \hline \\ \hline $	$S^2(e_{10})$	2.07	1.61	2.71	-0.35	6.04				
Change in the contribution to the variance in e_A of $S^2(e_C)$ 2.612.054.560.139.35 $S^2(A')$ 0.02-0.01-0.000.000.01 $cov(A', e_C)$ -0.08-0.440.380.380.25MenS²(e_1)3.813.4911.866.5925.75 $S^2(e_C)$ 6.292.8017.5616.5743.21 $S^2(e_A)$ 5.692.1813.6110.7932.26Change in the contribution to the variance in e_C of $S^2(e_1)$ 3.813.4911.866.5925.75 $S^2(e_D)$ 0.48-0.510.541.622.13cov(e_{10}, SD)2.00-0.185.168.3515.34Change in the contribution to the variance in e_A ofS²(e_C)6.292.8017.5616.5743.21S²(e_D)0.48-0.510.541.622.13Change in the contribution to the variance in e_A ofS²(e_C)6.292.8017.5616.5743.21S²(e_C)6.292.8017.5616.5743.21S²(e_C)6.292.8017.5616.5743.21S²(e_C)6.292.8017.5616.5743.21S²(e_C)6.292.8017.5616.5743.21S²(e_C)	$S^2(SD)$	-0.02	-0.12	0.20	0.12	0.19				
$\begin{array}{c c c c c c c c c c c c c c c c c c c $	$cov(e_{10},SD)$	0.56	0.56	1.64	0.36	3.12				
$\begin{array}{c c c c c c c c c c c c c c c c c c c $		Change in th	e contributio	on to the varia	ance in e_A of					
$\begin{array}{c c c c c c c c c c c c c c c c c c c $	$S^2(e_C)$	2.61	2.05	4.56	0.13	9.35				
Men $S^2(e_{10})$ 3.81 3.49 11.86 6.59 25.75 $S^2(e_C)$ 6.29 2.80 17.56 16.57 43.21 $S^2(e_A)$ 5.69 2.18 13.61 10.79 32.26 Change in the contribution to the variance in e_C of $S^2(e_{10})$ 3.81 3.49 11.86 6.59 25.75 $S^2(e_{10})$ 3.81 3.49 11.86 6.59 25.75 $S^2(e_{10})$ 3.81 3.49 11.86 6.59 25.75 $S^2(SD)$ 0.48 -0.51 0.54 1.62 2.13 $cov(e_{10}, SD)$ 2.00 -0.18 5.16 8.35 15.34 Change in the contribution to the variance in e_A of S^2(e_C) 6.29 2.80 17.56 16.57 43.21 S^2(e_C) 6.29 2.80 17.56 16.57 43.21 S^2(A') -0.01 0.10	$S^2(A')$	0.02	-0.01	-0.00	0.00	0.01				
$\begin{array}{ c c c c c c c }\hline S^2(e_{10}) & 3.81 & 3.49 & 11.86 & 6.59 & 25.75 \\ \hline S^2(e_C) & 6.29 & 2.80 & 17.56 & 16.57 & 43.21 \\ \hline S^2(e_A) & 5.69 & 2.18 & 13.61 & 10.79 & 32.26 \\ \hline Change in the contribution to the variance in e_C of \hline S^2(e_{10}) & 3.81 & 3.49 & 11.86 & 6.59 & 25.75 \\ \hline S^2(SD) & 0.48 & -0.51 & 0.54 & 1.62 & 2.13 \\ \hline cov(e_{10},SD) & 2.00 & -0.18 & 5.16 & 8.35 & 15.34 \\ \hline Change in the contribution to the variance in e_A of \hline S^2(e_C) & 6.29 & 2.80 & 17.56 & 16.57 & 43.21 \\ \hline S^2(A') & -0.01 & 0.10 & 0.21 & 0.41 & 0.70 \\ \hline \end{array}$	$cov(A', e_C)$	-0.08	-0.44	0.38	0.38	0.25				
$S^2(e_C)$ 6.29 2.80 17.56 16.57 43.21 $S^2(e_A)$ 5.69 2.18 13.61 10.79 32.26 Change in the contribution to the variance in e_C of $S^2(e_{10})$ 3.81 3.49 11.86 6.59 25.75 $S^2(SD)$ 0.48 -0.51 0.54 1.62 2.13 $cov(e_{10},SD)$ 2.00 -0.18 5.16 8.35 15.34 Change in the contribution to the variance in e_A ofS²(e_C) 6.29 2.80 17.56 16.57 43.21 $S^2(e_C)$ 6.29 2.80 17.56 16.57 43.21 $S^2(A')$ -0.01 0.10 0.21 0.41 0.70			Me	en						
$S^2(e_A)$ 5.692.1813.6110.7932.26Change in the contribution to the variance in e_C of $S^2(e_{10})$ 3.813.4911.866.5925.75 $S^2(SD)$ 0.48-0.510.541.622.13cov(e_{10},SD)2.00-0.185.168.3515.34Change in the contribution to the variance in e_A of $S^2(e_C)$ 6.292.8017.5616.5743.21 $S^2(A')$ -0.010.100.210.410.70	$S^2(e_{10})$	3.81	3.49	11.86	6.59	25.75				
Change in the contribution to the variance in e_C of $S^2(e_{10})$ 3.813.4911.866.5925.75 $S^2(SD)$ 0.48-0.510.541.622.13cov(e_{10},SD)2.00-0.185.168.3515.34Change in the contribution to the variance in e_A of $S^2(e_C)$ 6.292.8017.5616.5743.21 $S^2(A')$ -0.010.100.210.410.70	$S^2(e_C)$	6.29	2.80	17.56	16.57	43.21				
$ \begin{array}{ c c c c c c c } \hline S^2(e_{10}) & 3.81 & 3.49 & 11.86 & 6.59 & 25.75 \\ \hline S^2(SD) & 0.48 & -0.51 & 0.54 & 1.62 & 2.13 \\ \hline cov(e_{10},SD) & 2.00 & -0.18 & 5.16 & 8.35 & 15.34 \\ \hline \hline Change in the contribution to the variance in e_A of \\ \hline S^2(e_C) & 6.29 & 2.80 & 17.56 & 16.57 & 43.21 \\ \hline S^2(A') & -0.01 & 0.10 & 0.21 & 0.41 & 0.70 \\ \hline \end{array} $	$S^2(e_A)$	5.69	2.18	13.61	10.79	32.26				
$\begin{tabular}{ c c c c c c c c c c c c c c c } \hline S^2(SD) & 0.48 & -0.51 & 0.54 & 1.62 & 2.13 \\ \hline cov(e_{10},SD) & 2.00 & -0.18 & 5.16 & 8.35 & 15.34 \\ \hline \hline Change in the contribution to the variance in e_A of $$ $$ $$ $$ $$ $$ $$ $$ $$ $$ $$ $$ $$$		Change in the contribution to the variance in e_C of								
$\begin{tabular}{ c c c c c c c c c c c c c c c c c c c$	$S^2(e_{10})$	3.81	3.49	11.86	6.59	25.75				
Change in the contribution to the variance in e_A of $S^2(e_C)$ 6.292.8017.5616.5743.21 $S^2(A')$ -0.010.100.210.410.70	$S^2(SD)$	0.48	-0.51	0.54	1.62	2.13				
$S^2(e_C)$ 6.292.8017.5616.5743.21 $S^2(A')$ -0.010.100.210.410.70	$cov(e_{10},SD)$	2.00	-0.18	5.16	8.35	15.34				
$S^2(A')$ -0.01 0.10 0.21 0.41 0.70	Change in the contribution to the variance in e_A of									
	$S^2(e_C)$	6.29	2.80	17.56	16.57	43.21				
	$S^2(A')$	-0.01	0.10	0.21	0.41	0.70				
$cov(A', e_C) \qquad -0.59 \qquad -0.71 \qquad -4.17 \qquad -6.19 \qquad -11.65$	$cov(A', e_C)$	-0.59	-0.71	-4.17	-6.19	-11.65				

*2010 or latest available year

Table 5: Changes in the inequalities in the life-table summary measures based on statistics invariant to relative change in developed countries (per 1000 population). Total change in the inequalities between 1970 and 2010 (per 1000 population)

Statistic	1970-1980	1980-1990	1990-2000	2000-2010	1970-2010*			
Women								
$G(e_{10})$	4.6	2.5	3.2	-1.3	9.0			
$G(e_C)$	5.6	3.3	4.3	-1.3	11.8			
$G(e_A)$	5.8	3.0	4.7	-1.1	12.4			
	Change in th	e contributio	on to $G(e_C)$ of	•				
e_{10}	5.3	2.8	3.4	-1.6	10.0			
Δ	0.3	0.5	0.8	0.3	1.8			
	Change in th	e contributio	on to $G(e_A)$ of	•				
e_{10}	5.5	2.9	3.5	-1.6	10.3			
Δ	0.2	0.4	0.8	0.3	1.8			
A'	0.0	-0.3	0.3	0.3	0.3			
		Men						
<i>e</i> ₁₀	6.6	4.2	8.9	2.0	21.7			
$G(e_C)$	8.7	4.8	11.1	1.9	26.5			
$G(e_A)$	8.5	4.3	10.1	2.1	25.0			
Change in the contribution to $G(e_C)$ of								
Total contribution of e_{10}	7.6	4.6	9.9	2.0	24.1			
Total contribution of Δ	1.1	0.1	1.2	0.0	2.4			
Change in the contribution to $G(e_A)$ of								
e _C	7.9	4.7	10.3	2.0	24.8			
Δ	1.0	0.2	1.1	0.0	2.3			
A'	-0.3	-0.6	-1.3	0.2	-2.1			

*2010 or latest available year

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