Analyzing the Trends of the Modal Age at Death for European Countries and Japan Using the LD Model

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Introduction

Recently, the modal age at death has been paid more attention as an indicator of longevity (Horiuchi et al. 2013). Although many studies have discussed the modal age at death, there have been few articles that examined decomposition analyses of the change of the modal age in terms of the shifting and/or the compression of the mortality curve.

The author has proposed the Linear Difference (LD) model that is a shift-type adult mortality model and shown that the model has some advantages for the modeling of adult mortality for Japan and several EU countries compared to the decline-type model such as the Lee-Carter model (Ishii and Lanzieri 2013).

In this paper, we propose a new decomposition method for the modal age at death using the LD model, and give decomposition analyses with the method for several European countries and Japan.

1 Data and Method

In this paper, we use mortality data for female from 1970 to 2010 by the Human Mortality Database(HMD) and those by the Japanese Mortality Database (JMD) for Japan. We use the following notations used in Ishii and Lanzieri (2013).

$y = \lambda_{x,t} = \log \mu_{x,t}$: log hazard function of mortality

$\nu_{y,t}$: the inverse function of $y = \lambda_{x,t}$ (if defined)

$$\rho_{x,t} \underset{\text{def}}{=} -\frac{\partial \lambda_{x,t}}{\partial t} = -\frac{\partial \log \mu_{x,t}}{\partial t} \text{ : the mortality improvement rate}$$

$$\tau_{y,t} \underset{\text{def}}{=} \frac{\partial \nu_{y,t}}{\partial t} \text{ : the force of age increase}$$

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Let us define the LD model satisfying the property that $\tau_{y,t}$ is a linear function of x for each t, i.e. $\tau_{y,t} = f'_t + g'_t x$. By integrating both sides with t, we obtain $\nu_{y,t} = f_t + g_t x + a_y$ where a_y denotes a standard pattern of inverse log hazard rates.

Figure 1　Stylized example of the LD model

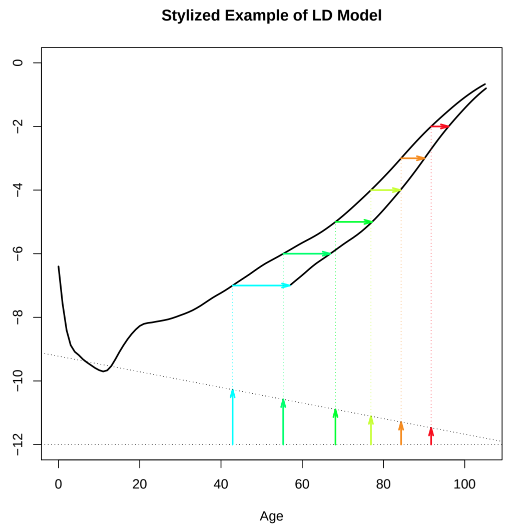
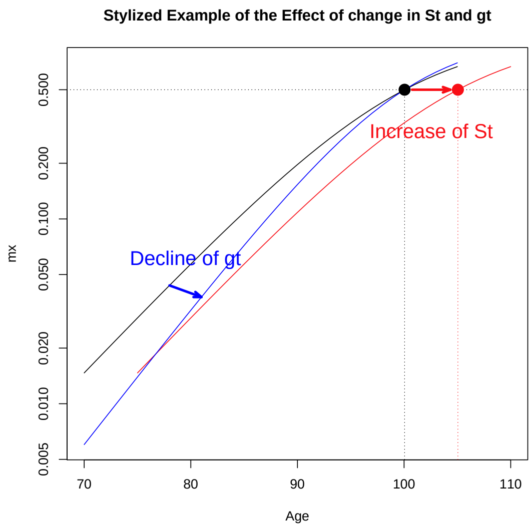


Figure 2　Stylized example of the LD model



The Figure 1 shows the stylized example of the LD model. The colored horizontal arrows in the upper half of the Figure 1 show the amount of shifts of the mortality curve indicated with the black line, which correspond to the $\tau_{y,t}$. The vertical arrows at the bottom have same lengths as in the upper side with the same color whose directions are rotated 90 degrees counter-clockwise. The LD model requires that the amount of shifts is a linear function of age, which means the end point of the arrows form a line. The parameter g'_t means the slope of the above line, so the g_t means the slope of age increases between time t and t_0 (base point of time).

Here, we consider another variable S_t as a location of the mortality curve instead of f'_t or f_t. S_t is defined as the age that the mortality rate equals to 0.5 at time t. We can always covert from S_t to f_t using the value of g_t. The Figure 2 shows the stylized example of the effect of change in S_t and g_t. Assuming that the mortality curve at a base point of year is shown as the black line, the increase of S_t with g_t fixed changes the curve into one shown as the red line. Therefore, we can recognize the mortality improvement by the increase of the S_t as the shifting of the mortality curve. On the other hand, the decline of g_t with S_t fixed changes the curve into one shown as the blue line, which exhibits some compression features of mortality during the improvement.

In the LD model, we can derive the following decomposition of the trends of M_t: the modal age at death.

$$\frac{d}{dt}M_t = f_t' + g_t' \left(M_t - \frac{1}{\mu_{x,t} - \frac{\frac{\partial^2}{\partial x^2}\lambda_{x,t}}{\frac{\partial}{\partial x}\lambda_{x,t}}} \right) = S_t' + g_t'(M_t - S_t) - g_t' D_t$$

where $D_t = \frac{1}{\mu_{x,t} - \frac{\frac{\partial^2}{\partial x^2}\lambda_{x,t}}{\frac{\partial}{\partial x}\lambda_{x,t}}}$.

This formula is interpreted as follows. The S_t' stands for the amount of shifting, the $g_t'(M_t - S_t)$ does for the effect of compression at the modal age, and the $-g_t' D_t$ does for the gap of the modal age at $t + dt$ and the age at $t + dt$ that the value of the $\lambda_{x,t}$ of the modal age at t is taken. Moreover, the formula $\frac{d}{dt}M_t = f_t' + g_t'(M_t - D_t)$ could also be seen as the change of the modal age at death is equal to the force of age increase for the age $M_t - D_t$.

2 Results

Ishii and Lanzieri (2013) has shown that the LD model works very well for some EU countries, whereas it does not much for others. Here, we show the results for Japan, France, West Germany and Austria that the LD model exhibits good fit.

Figure 3 shows the trends of the M_t for the actual mortality and the LD model.[*1] We can observe that the both trends are similar, so we will analyze with the mortality rates by the LD model.

Figure 4 shows the results of the decomposition results of the change of M_t for the LD model by every ten years for Japan. We can observe that the increase of M_t is mainly caused by the shifting from 1980 to 2000, whereas compression plays a larger part before 1970 and after 2000. Figure 5 shows the results for France, West Germany and Austria.

References

Canudas-Romo, V. (2008) “The modal age at death and the shifting mortality hypothesis”, *Demographic Research*, Vol. 19, No. 30, pp. 1179–1204.

Horiuchi, S., N. Ouellette, S. L. K. Cheung, and J.-M. Robine (2013) “Modal Age at Death: Lifespan Indicator in the Era of Longevity Extension”, Paper presented at the XXVII IUSSP International Population Conference.

Human Mortality Database. University of California, Berkeley (USA) and Max Planck

[*1] To calculate the M_t, we used the approximation by quadratic function originally proposed by Kannisto and used in Canudas-Romo (2008).

Figure 3 Trends of the Modal Age at Death (Actual and LD, Female)

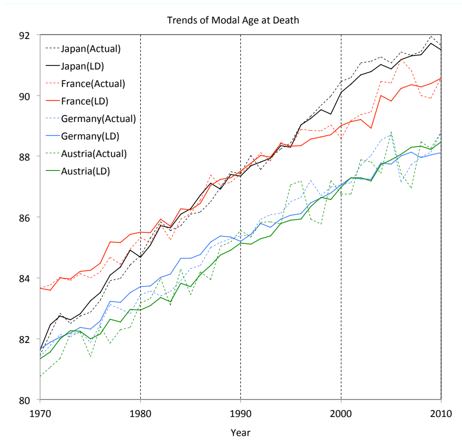


Figure 4 Decomposition of the change of Modal Age at Death (Female Japan)

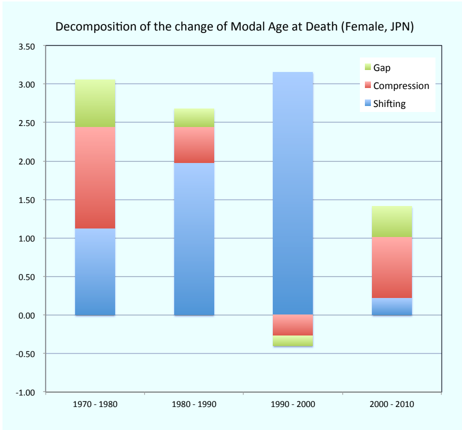
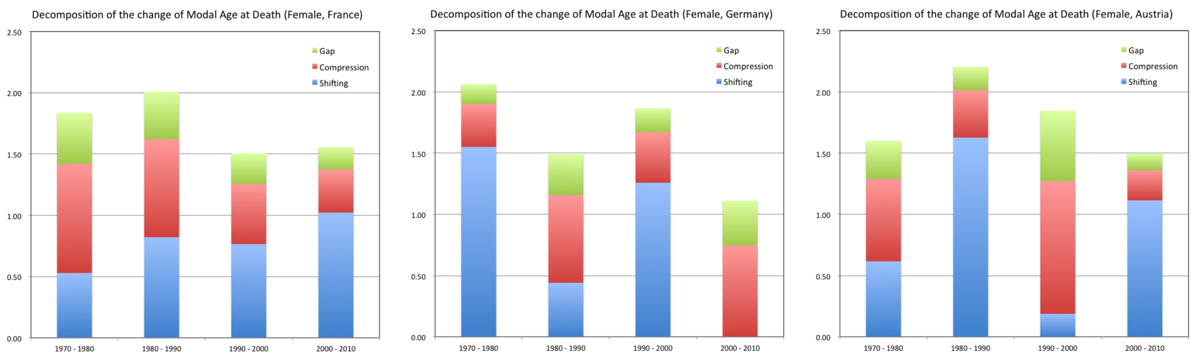


Figure 5 Decomposition of the change of Modal Age at Death (Female, France, West Germany and Austria)



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