Differential Game of Pollution Control with overlapping generations

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Extended Abstract

Environmental issues are a classical topic where game theory and differential games are used to study the strategic interaction between agents (i.e. individuals). Many papers in this field use players that are symmetric in their age and in their duration of the game. This is a significant restriction to the model because of the following reasons. i) If a model uses real persons as players it is a considerable restriction to ignore the survival schedule of the persons. Individuals are born, age, die and their life-horizon is finite. Further, the set of players who are acting is age-structured. So players act against players in all other age groups. Moreover the preferences of the players possibly change over their age. Thus it is straightforward to assume that the strategies of the players will change according to their age. ii) If a model uses representative players with infinite time horizon the players will include

the whole time horizon (no finite restriction to time) in their optimization. If a model on the other hand uses representative players with finite time horizon (the life-cycle is finite) the players will pollute the environment more when they are nearing their end of life. However, in reality the story lies in between. The agents should have a finite life, but the environment exists forever.

To overcome the above two points we use an overlapping generations framework with continuous age-structure. That allows to include the finite life-time of the agents who act over their own life-cycle, but pollute the environment which evolves over time. With this simple model we can address a very important topic. Many players act over a finite life-time and pollute the environment. Therefore later living cohorts will suffer from that *myopic* behavior of earlier cohorts. To account for the fact that real persons also care about their own offsprings and have a positive utility of their well-being, we introduce an altruistic motive. This is modeled in the established Barro-Becker-style (see [1]) and allows to see whether the altruistic motive is enough to overcome the opposite interests of the finite time horizon and environmental pollution over an infinite time horizon. Finally we illustrate and provide an interpretation of the difference between the optimal strategies in the non-cooperative and the cooperative solution (i.e. the social welfare). The result shows a fundamental difference between the two solutions and that the altruistic motive decreases optimal emissions, but cannot turn the result into the cooperative one.

To the best of our knowledge the first differential game model using an overlapping generation structure is [5] in a renewable resource extraction context. Their model uses an overlapping generation framework where new cohorts enter the model at discrete points in time. The players exploit a stock of renewable resources.

Our model highlights two differences compared to [2], [3]. Firstly, taxes in emissions in our model does not depend on the stock of emissions. This is due to the linear form of the damage function in the objective function, which has been estimated in [6] and also used in [4]. Secondly, in our model the tax rate depends on the age of the individual. This is driven by including overlapping generations and finite lives into the model. At each point in time the generations face a different finite time horizon, thus they act differently. Consequently it is not possible to reach the socially optimal outcome with an age-independent outcome.

Futhermore we derive an time-consistent age-dependent tax on emissions to overcome the problems of the asynchronous time horizon. Note that this result is important from a political point of view. Nowadays emissions and the resulting climate change is a big issue, but there are many different opinions around how to reduce emissions on a total level. Our small model can be seen as a very simple model on climate change, where, $e(\cdot)$ denote CO2 emissions, modeled as the optimal choice for each individual. $P(t)$ denotes the corresponding stock in the athmosphere, modeled dynamically by

$$\dot{P}(t) = E(t) - \delta P(t), P(0) = P_0 \tag{1}$$

where $E(t)$ are the aggregated CO2-emissions of all living individuals (accross all cohorts) at time t and δ is the rate by which the environment regenerates. The crucial message is that the emissions will be much higher when every player maximizes only over the own life compared to a cooperative solution. Even including altuism into the model cannot solve the problems of the myopic non-cooperative solution. However, it is possible to introduce a tax on emissions that turns the non-cooperative result into the cooperative one (in a subgame perfect way). The tax depends on age and time, which corresponds to the fact that emissions should (in a socially optimal way) be constant across age but increasing over time, but are the other way around (i.e. increasing over the life-cycle, but constant over time for every age-group) in the cooperative solution.

The model can be extended in a couple of directions. First, the damage of the stock of pollution should be allowed to be a general (non-linear) function. Convex, concave as well as other forms are possible. Second, the cooperative solution should be extended to the infinite time horizon. In this case it is interesting to derive the condition and the level of a steady state. Further the difference to the above case with the finite time horizon is interesting and will propose important conclusions from a political point of view (politicians act usually up to a finite time horizon, e.g. the average temperature should not increase by more the 2 degree in the next century). Third, it is realistic to assume that the stock of pollution influences the health of the players. i.e. the survival probability as well as the fertility rate should depend on $P(t)$. This is interesting because of the interpretation, as well as from a game theoretic point of view, since the strategies of the players influence the number of players in the future (during and after the own life). Fourth, it seems to be realistic that there is a possibility to invest into environmental protection (e.g. R&D in CO2 reduction technology). This possiblity probably shows a further crucial difference between the non-cooperative and the cooperative solution.

Finally, it is very interesting to introduce a second type of player (i.e. the government) who fixes the taxes. This player then has a different time horizon than the other players (and a different objective function). In the current model we have derived the taxes such that the non-cooperative result equals that of the cooperative sulution. In the case where the government fixes the taxes, it will also depend on their objective function. Also this extension is interesting from a methodological point of view.

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