Cohort mortality forecasting: examples from selected European countries

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Introduction

Study of “mortality laws” (or general mortality patterns) is one of the fundamental demographic issues already from the beginnings of demography itself. It could be followed up to 17th century and John Grount’s study of Bills of mortality in London (Graunt, 1662).

In the contemporary world the need for accurate mortality analyses and forecast is supported among others also by its connection to life insurance products or social policy. In many spheres of life information about mortality development and its expected future changes become more and more crucial.

The aim of the paper

The aim has three parts: one is to introduce an alternative (alternative to the traditional ones) method of mortality estimation and forecast focused on higher (adult) ages which is based on cohort approach and could be used for estimation of intensity of mortality for not yet extinct cohorts. The second part of the aim is to evaluate the proposed method using data from Sweden (described below) and other selected European countries where the data time-series is long enough. Estimation of cohort life expectancy based on presented model for already extinct cohorts will be compared with empirical life expectancy which is already known. The third part will be forecasting of cohort life expectancy for still living cohorts in selected European countries will be done.

Some of the traditional approaches to mortality modelling

In recent years the method of Lee-Carter is very popular and almost generally used. It is predictive method of estimation the future development of age-specific mortality rates (Lee, Carter, 1992). In its original form it combines factor of the average level of mortality for any certain age (time invariant), factor of general mortality development (its changes in time) and age-specific constant expressing the age-specific rate of mortality change when general mortality level is changing (*ibid*.). It can be concluded that this method in its basic form is based on forecast of period age-specific mortality rates to any time point in the future, then any other measure could be derived – e.g. the life expectancy.

We can find many modifications of the Lee-Carter method because for many studies the combination of influence of age and time was nearly insufficient. As a result also the “cohort effect” was included. One of the most recent and most important modifications of the Lee-Carter method (using also the cohort effect) are works of Renshaw and Haberman (Renshaw, Haberman, 2006).

Usually all methods, defined in a regression form, need sophisticated statistical software to be solved. Enrichment of the models of additional factors leads to the increase in the number of parameters to be estimated and the final model can contain up to hundreds of variables (e.g. Alai, Sherris, 2012). Logically, dealing with such a model is highly technically demanded.

Approaches not working with values of age-specific mortality rates but with some relations of the rates are other special type of mortality models. Usually those relations of mortality rates are ratios or differences of the two consecutive rates and its development in time (development of those ratios). For example in the work of Haberman and Renshaw the age-specific differences of mortality rates across time

are modeled. These differences are called “mortality improvements” and so the “rate of mortality improvement” is estimated for particular age groups also in the future (Haberman, Renshaw, 2013).
Another approach could be based on modeling of ratios (or differences) of mortality rates between two ages (age groups) in time. Such an analysis was published by Mazouch (2010), however, this approach has many weaknesses. Modeling of ratios is included also in works of Börger and Aleksic (2013), Mazouch and Hulíková Tesárková (2010 and 2012 – more explained below).

Data and methods used in the paper

For illustration of the method data from the Human Mortality Database (www.mortality.org) were used. The longest time series in the Database is available for Sweden (starting in 1751, cohort data are available starting for the cohort 1676 to cohort 1981). For the analysis data for ages 60 and more were used.
The proposed method is based on the relationship:

$$ar_{x,z} = \frac{m_{x,z}}{m_{x-1,z}},$$

where $ar_{x,z}$ is **ratio of age-specific mortality rates defined for one particular cohort**, *x* is completed age, *z* is the year of birth of the cohort. Because these ratios are highly time-variant (differs for particular cohorts, however, it was proved that its values do not follow any particular trend), it was necessary to estimate these ratios for individual ages in a particular modeled cohort by some average measure calculated for that age ($\overline{ar}_{x,z,n,s}$). This measure was calculated from *n* previous cohorts and these average measures are used for the estimation of future values of age-specific mortality rates. There are many possibilities how to estimate $\overline{ar}_{x,z,n,s}$. Among others it is possible to use the relation:

$$\overline{ar}_{x,z,n,s} = \frac{\sum_{k=1}^{n} \alpha^{k-1} * ar_{x,z-k-x+s}}{\sum_{k=0}^{n} \alpha^{k-1}}, \text{ for } x = s, \ldots, \omega - 1$$

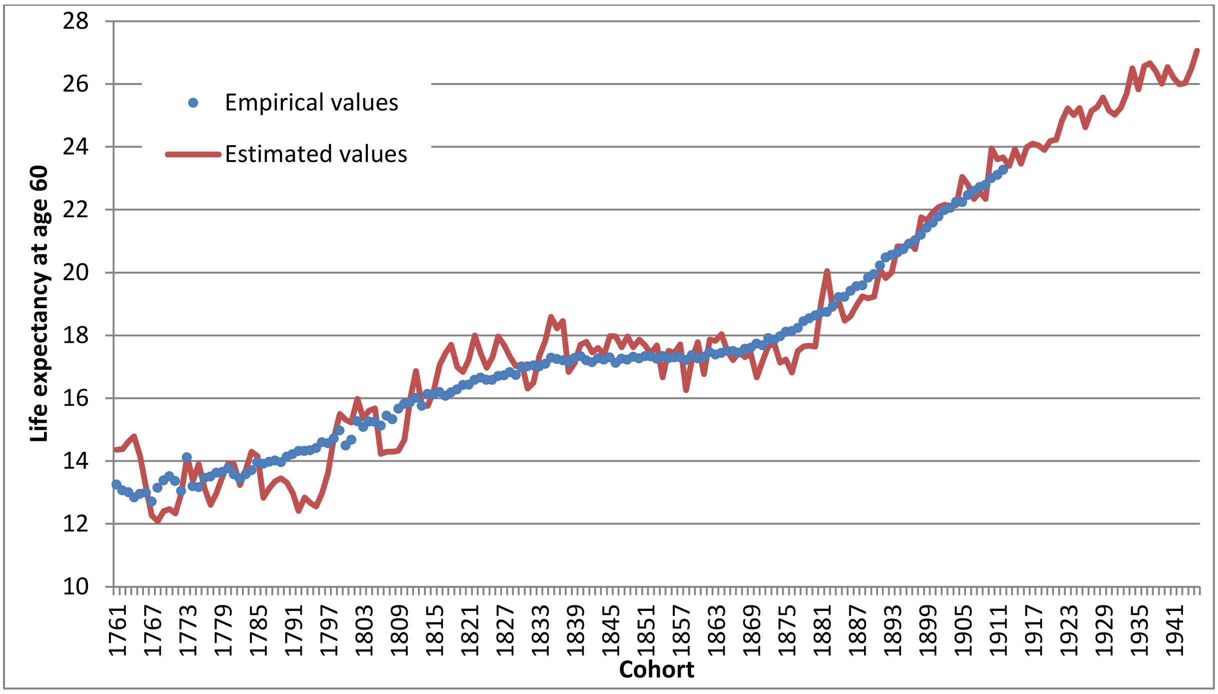
where $\overline{ar}_{x,z,n.s}$ is the **average ratio of cohort-specific rates** at age *x*, *z* is the year of birth of the modeled cohort, *s* is first age which is used for calculation (for example 60 years), α is weight used in the model having values from the interval (0;1). This weight could be selected subjectively according to needed “memory“ of the model.
After all the average ratios are calculated for all ages (*x*) where mortality will be estimated, it is possible to calculate the first unknown mortality rate (for age *x + 1*):

$$\dot{m}_{x+1,z} = m_{x,z} * \overline{ar}_{x+1,z,n,s}$$

Results

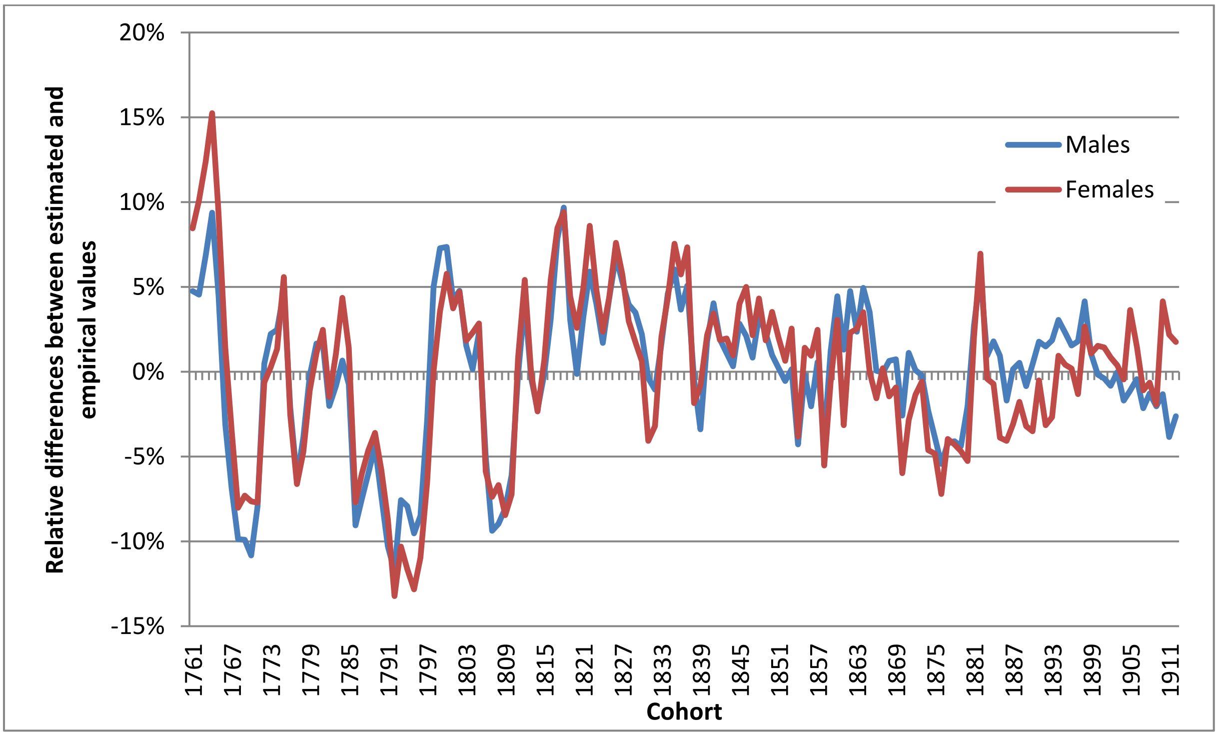
Quality of the model is verified by comparison of estimated values and empirical values of already extinct cohorts. Cohort life expectancy at age 60 for Swedish females were at the beginning of analyzed period, for cohort born in 1761, 13 years, one year higher than for males. In next 150 cohorts the life expectancy for females grew to 23,3 for females and to 18,5 for males. The development of the life expectancy was not linear among analyzed cohorts (cohorts 1761-1912), there were some periods of stagnation and periods of acceleration (data from www.mortality.org; see Fig. 1 below).
It was proved empirically, that the model described above respects all those changes in trend of life expectancy and estimated values are not dissimilar to empirical values. Differences between estimates and empirical values were decreasing in their values for younger cohorts. The biggest differences for males occurred for cohort born in 1752, modeled life expectancy at age 60 was underestimated for 1.5 years. For females was the biggest difference the overestimation for 1.9 years for cohort born in 1764.

Fig. 1: Comparison of empirical and modeled life expectancy at age 60, cohorts born 1761–1945, Sweden, females



Source: Human Mortality Database (2012), author's calculation

Fig. 2: Relative differences between estimated and empirical values of life expectancy at age 60, cohorts 1761–1912, Sweden, in %



Because of increasing values of life expectancy in time and decreasing differences between estimated and empirical values of life expectancy we can calculate the relative differences for evaluation of the model (Fig. 2) as:

$$\frac{\dot{e}_{x,z}}{e_{x,z}} - 1,$$

where $e_{x,z}$ is empirical cohort life expectancy at age *x* for cohort *z* and $\dot{e}_{x,z}$ is estimated cohort life expectancy at age *x* for cohort *z*.

Results are in Fig. 2 where we can see decreasing values of relative differences. With the exception of the first highly volatile period, values of the ratio are almost stable and not higher than 5 percent (over or underestimated).

Conclusion

It is necessary to keep in mind that mortality patterns are very complex and depend on many other factors (cohort effect, period effect, age effect) and to find and describe all those relationships is almost impossible.

The above described model is very simple way how to estimate cohort mortality development in the future. Its advantage could be low demands for software or any other technical equipment and mathematical knowledge. It works with very simple presumptions and its application is not time-consuming in comparison to the other common models which are used.

Results show that estimated values are very close to empirical ones and that also changes in trends were estimated correctly. That makes this method promising for cohort mortality analyses as well as in cases where future mortality of not yet extinct cohorts should be estimated.

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