Coherently forecasted mortality and its sensitivity to the method and the selection of countries

Lenny Stoeldraijer (Statistics Netherlands and Population Research Centre (University of Groningen), the Netherlands)

Fanny Janssen (Population Research Centre, University of Groningen, the Netherlands)

DRAFT. PLEASE DO NOT QUOTE WITHOUT PERMISSION OF THE AUTHORS

Abstract

In Western Europe, convergence has been observed in old-age mortality, and convergence in mortality between countries is likely thanks to common socio-economic policies, similar progress in medical technology, and shared importance of certain life style factors over time. However, mortality forecasts for single populations will in the long run lead to divergent outcomes, contrary to what would be expected based on historical trends. Therefore, different coherent forecasting methods are introduced, in which “coherent” refers to non-divergent forecasts for subpopulations within a larger population. A crucial question is the identification of the countries which determine the basic mortality trend that will be applied to other countries.

The purpose of this research is to analyse the sensitivity of projected mortality based on different coherent forecasting methods and the sensitivity of the outcome due to the selection of the countries to take as a group.

Data from several countries on all-cause mortality and population numbers by sex, age (0, 1-4, 5-9, …, 90-94, 95+), and year (1950-2009) will be obtained through the Human Mortality Database. The different coherent forecasting methods are the Li-Lee method and a coherent functional method. We will make out-of-sample forecasts up to 2050 to test the sensitivity of the method and choice of countries, and within-sample forecast to see how accurate the results will be.

We expect that mortality forecasts for countries with historically a more linear mortality trend are more accurate, regardless which countries are taken as a group. For countries with less linear trends, we expect an important impact of the choice of the group of countries.

1. Introduction

With the rapid population aging, mortality forecasting is becoming important. Mortality forecasts are valuable for social security programs and are often used to predict the sustainability of pension schemes. Forecasts of future mortality levels, especially among the elderly, are important in enabling governments to provide for health and other needs of their societies.

The importance of mortality forecasts has resulted in the development of numerous models for mortality modelling and forecasting (for reviews see Pollard 1987; Tabeau 2001; Wong-Fupuy and Haberman 2004; and Booth and Tickle 2008). The majority of these methods can be classified under the extrapolative methods, which make use of the regularity typically found in both age patterns and trends over time, with the Lee-Carter method (Lee and Carter 1992) becoming the dominant method of mortality forecasting. The Lee-Carter method summarizes mortality by age and period for one single population into an overall time trend, an age component, and the extent of change over time by age (Lee and Carter 1992). It forecasts probability distributions of age-specific death rates using standard time series procedures.

One of the strengths of the Lee-Carter method, and extrapolation methods more in general, is its robustness in situations where age-specific log mortality rates have linear trends (Booth et al. 2006). However, there are examples of countries which have less linear trends, such as the Netherlands, Denmark and Norway. If the trend is not linear, the forecasted mortality could be very different depending on the fitting period (Stoeldraijer et al. 2013).

Another important issue is that mortality forecasts using extrapolation methods based on information of each country separately, might result in divergence, contrary to historic trends. In Western Europe, convergence has been observed in old-age mortality (Janssen, Mackenbach and Kunst 2004), and convergence between countries is likely thanks to common socio-economic policies, similar progress in medical technology, and shared importance of certain life style factors over time. Furthermore, it is likely that mortality levels of countries with similar mortality evolutions continue to evolve in parallel.

To overcome the latter issue, coherent forecasting methods are introduced, where “coherent” refers to non-divergent forecasts for sub-populations within a larger population (Li and Lee 2005). The idea behind coherent forecasting is that mortality forecasts for countries with similar mortality developments will not diverge radically, but also that structural differences between groups will remain, such as the consistently higher mortality for men than for women. An additional benefit of coherent forecasting is that the more linear trends for some countries might provide better information about the future direction of the mortality trends in other countries with less linear trends. The experiences in other countries can thus be used to create a broader empirical basis for the identification of the most likely long-term trend.

A crucial question in any coherent forecasting method is which countries will determine the central tendency, or basic mortality trend, that will be applied to other countries (henceforth the main group). Li and Lee (2005) concluded that using a larger group of similar populations is useful in assessing the mortality trend for an individual country. A more formal analysis, however, has not been performed yet, whereas differences can be expected to be large.

Until now, to our knowledge, two main coherent forecasting methods have been proposed in the international scientific literature. Li and Lee (2005) extended the Lee-Carter model (Lee and Carter 1992) by taking into account the mortality experiences in other countries. Short-term differences in mortality are preserved, but in the long term, the age-specific death rates within the group of countries are limited to a constant ratio to one another. The coherent functional data (CFD) method of Hyndman, Booth and Yasmeen (2013) involves forecasting interpretable product and ratio functions of rates using the functional data paradigm introduced in Hyndman and Ullah (2007). These methods are very different and their outcomes have – so far - not yet been compared with each other.

The purpose of this research is to analyse the sensitivity of forecasted mortality based on (i) the two abovementioned main coherent forecasting methods and (ii) the selection of the main group. We will do so for France and the Netherlands.

We expect that mortality forecasts for countries with historically a more linear mortality trend (like France) are more accurate and less sensitive, regardless which main group is taken. For countries with less linear trends (such as the Netherlands), we expect an important impact of the choice of the main group.

2. Data & Methodology

2.1 Data

Data on all-cause mortality and population numbers by sex, age (0, 1-4, 5-9, …, 90-94, 95+), and year (1970-2009) were obtained through the Human Mortality Database (accessed January 16, 2014). All countries, spanning the period 1970-2009, from the Human Mortality Database were included in the analysis, except for Iceland and Luxembourg because of their small numbers and zero deaths for some age groups. This results in a total of 30 countries.

2.2 Coherent forecasting methods

2.2.1 The Li-Lee method

The Li-Lee method (Li and Lee 2005) is an extension of the Lee-Carter method (Lee and Carter 1992). In essence, the Lee-Carter method is applied twice: first to all populations combined, and then to the residuals.

The model for all populations combined is given by

$$\ln\left(M_{x,t}\right) = A_x + B_x K_t + E_{x,t} \tag{1}$$

where $M_{x,t}$ denotes the death rate at age x and year t of all populations combined, A_x equals the average over time of $\ln\left(M_{x,t}\right)$, B_x is the set of age-specific constants that describe the relative rate of change at any age, K_t denotes the underlying time development and $E_{x,t}$ the residual error. B_x and K_t are found using Singular Value Decomposition under the assumptions $\sum_x B_x = 1$ and $\sum_t K_t = 0$. After estimation, K_t is adjusted to fit the observed life expectancy (Lee and Miller 2001) and extrapolated using a random walk with drift.

The model for the residuals is given by

$$\ln\left(m_{x,t,i}\right) - a_{x,i} - \hat{B}_x \hat{K}_t = b_{x,i}^{res} k_{t,i}^{res} + \varepsilon_{x,t,i}^{res} \tag{2}$$

where $m_{x,t,i}$ denotes the death rate of population i, $a_{x,i}$ equals the average over time of $\ln\left(m_{x,t,i}\right)$ and $\hat{B}_x \hat{K}_t$ are the estimates from the first equation. $b_{x,i}^{res}$ is the set of age-specific constants that describe relative rate of change at any age, $k_{t,i}^{res}$ denotes the underlying time development and $\varepsilon_{x,t,i}^{res}$ the residual error. Again, Singular Value Decomposition is used to estimate $b_{x,i}^{res}$ and $k_{t,i}^{res}$. $k_{t,i}^{res}$ is extrapolated using an autoregressive model (AR(1) or a higher order model if $k_{t,i}^{res}$ does not converge to a constant).

The estimates are combined into one model for the population concerned:

$$\ln\left(m_{x,t,i}\right) = a_{x,i} + B_x K_t + b_{x,i}^{res} k_{t,i}^{res} + \varepsilon_{x,t,i} \tag{3}$$

2.2.2 The CFD method

The coherent functional data (CFD) method (Hyndman, Booth and Yasmeen 2013) can be viewed as a generalization of the Li-Lee method, with the difference that the CFD method use up to six principal components (B_x and $b_{x,i}^{res}$ are the first principal components of model (1) and (2)), more general extrapolation models and smoothing. It involves forecasting interpretable product and ratio functions of rates using functional time series models introduced in Hyndman and Ullah (2007).

First, the death rate $m_{x,t,i}$ for population i at age x and year t are smoothed using weighted penalized regression splines (Wood 1994):

$$\ln\left(m_{x,t,i}\right) = \ln\left(f_{x,t,i}\right) + \sigma_{x,t,i}\varepsilon_{x,t,i} \tag{4}$$

where $f_{x,t,i}$ is an underlying smooth function. $\sigma_{x,t,i}$ allows the amount of noise to vary with age x. Then the products ($p_{x,t}$) and ratios ($r_{x,t,i}$) of the smoothed rates for each population i are defined:

$$p_{x,t} = \left[\prod_{i=1}^{I} f_{x,t,i}\right]^{1/I} \quad \text{and} \quad r_{x,t,i} = f_{x,t,i} / p_{x,t} \tag{5}$$

These products and ratios are then modelled using functional time series models, which are estimated using the weighted principal components algorithm of Hyndman and Shang (2009):

$$\ln\left(p_{x,t}\right) = \mu_{p,x} + \sum_{k=1}^{K} \beta_{t,k}\phi_{x,k} + e_{x,t} \tag{6a}$$

$$\ln\left(r_{x,t,i}\right) = \mu_{r,x,i} + \sum_{l=1}^{L} \gamma_{t,l,i}\psi_{x,l,i} + w_{x,t,i} \tag{6b}$$

where $\mu_{p,x}$ and $\mu_{r,x,i}$ are the means of $p_{x,t}$ and $r_{x,t,i}$, respectively, $\phi_{x,k}$ and $\psi_{x,l,i}$ are the principal components obtained from decomposing $p_{x,t}$ and $r_{x,t,i}$, respectively, and $\beta_{t,k}$ and $\gamma_{t,l,i}$ are the corresponding principal component scores. $e_{x,t}$ and $w_{x,t,i}$ are the error terms.

Forecasts are obtained by forecasting each coefficient $\beta_{t,1}$, … , $\beta_{t,K}$ and $\gamma_{t,1,i}$, … , $\gamma_{t,L,i}$ independently. $\beta_{t,1}$, … , $\beta_{t,K}$ are forecasted using autoregressive integrated moving average (ARIMA) models. $\gamma_{t,1,i}$, … , $\gamma_{t,L,i}$ are forecasted using any stationary autoregressive moving average (ARMA) or autoregressive fractionally integrated moving-average (ARFIMA) process.

The implied model for each population is given by

$$\ln\left(f_{x,t,i}\right) = \ln\left(p_{x,t} r_{x,t,i}\right) = \mu_{p,x} + \mu_{r,x,i} + \sum_{k=1}^{K} \beta_{t,k}\phi_{x,k} + \sum_{l=1}^{L} \gamma_{t,l,i}\psi_{x,l,i} + e_{x,t} + w_{x,t,i} \tag{7}$$

2.3 Analysis

The Li-Lee method and the CFD method are applied to France and the Netherlands, producing out-of-sample mortality forecasts up to 2050 and within-sample mortality forecast for the period 1990-2009. We choose the Netherlands given the relatively non-linear past development in mortality and France given the relatively past linear mortality development. We use several selections of the main group of countries to perform the forecasts: a large group of countries, ‘similar’ countries (based on the life expectancy at birth in 2009), and close neighbours (Western Europe). The countries belonging to the three groups are:

- Group 1: all countries (Australia, Austria, Belarus, Belgium, Bulgaria, Canada, Czech Republic, Denmark, England & Wales, Estonia, Finland, France, Hungary, Ireland, Italy, Japan, Latvia, Lithuania, Netherlands, Norway, Poland, Portugal, Russia, Slovakia, Spain, Sweden, Switzerland, Taiwan, U.S.A., Ukraine).
- Group 2: ten countries with the highest life expectancy at birth in 2009 (men and women combined) (Australia, Canada, France, Italy, Japan, Netherlands, Norway, Spain, Sweden, Switzerland).

- Group 3: Western Europe (Austria, Belgium, Denmark, England & Wales, Finland, France, Ireland, Italy, Netherlands, Norway, Portugal, Spain, Sweden, Switzerland).

We use the observed values for 2009 as the jump-off rates for both methods. The forecasts are made for each sex independently and for the sexes combined. For reference purposes, individually estimated forecasts for France and the Netherlands are also included in the results. The individually estimated forecasts are made with the Lee-Carter method (Lee and Carter 1992, LC method) and independent functional time series models (Hyndman and Ullah 2007, FD method).

The forecasts are made with the program R. For the method of Hyndman, Booth and Yasmeen (2013) we used the Demography package for R (Hyndman 2010).

3. Results

For both the Netherlands and France, the life expectancy at birth increased over the period 1970-2009 (Figure 1). For France, the trend is almost a straight line, whereas for the Netherlands there are periods with a strong increase and periods with a moderate increase. The trend and level for France is similar to the trend and level for the countries in Western Europe (group 3) and to the trend for the ten countries with the highest life expectancy at birth (group 2), but the life expectancy for France lies below the life expectancy of group 2 over the whole period. The trend for the Netherlands is similar to the trend for all countries together (group 1), but at a higher level. The life expectancy in the Netherlands is higher than the life expectancy of group 2 in the 1970s and just below the life expectancy of group 3 in the later period.

Independent of which coherent forecasting method is used, the lower life expectancy of group 1 results in the lowest future life expectancy for France and the Netherlands when group 1 is used as the main group (Table 1). For women, and men and women together, using group 2 gives the highest future life expectancy, independent of the method. For men, using group 3 and the CFD method gives the highest future life expectancy. Using the Li-Lee method, the highest future life expectancy is given using group 2.

From the results of the Li-Lee forecasts (Figure 1a), it follows that when the trend of the group is similar to the development of the country (looking at the life expectancy at birth), the Li-Lee forecast is almost equal to a Lee-Carter forecast. For instance, for the Netherlands the trend in life expectancy at birth is similar to the trend in life expectancy at birth for group 1 (although on a lower level). The

Lee-Carter forecast for the Netherlands is almost equal to the Li-Lee forecast with group 1. The trend in France is similar to the trend of group 3, and the Lee-Carter forecast and Li-Lee forecast with group 3 are similar as well. The Li-Lee forecasts also show that a more positive past trend of the group compared to the country, leads to a higher future life expectancy, while a more positive past trend of the country compared to the group, leads to a lower future life expectancy. Furthermore, there is wider range in the future life expectancy at birth for France (2.74 years) compared to the Netherlands (2.23 years). This is surprising because the trend in life expectancy for France is more linear then for the Netherlands.

The range in future life expectancy at birth using the CFD model is much smaller than the range using the Li-Lee model (Figure 1b). For both France and the Netherlands, this does not hold for the future life expectancy for men: in this case, the range using the CFD model is wider than the range using the Li-Lee model (Table 1). Using the functional data model without coherence (FD model), the future life expectancy for France is similar to the future life expectancy for France using the CFD model (especially to the future life expectancy of group 3), but for the Netherland the future life expectancy using the FD model is clearly lower than the future life expectancy using the CFD model (Figure 1b).

There is a clear difference between the Li-Lee results and CFD results with respect to the age distribution. At higher ages the range in the death rate in 2050 is much smaller for the CFD forecasts than for the Li-Lee forecasts, while it is reversed at younger ages (Figure 2). Especially the range of the CFD forecasts at ages 0, 1, and 5 is high. There is also a clear increase in range between ages 25 and 35 for the CFD forecasts. The range for the Li-Lee forecasts is more equal over the ages, but the difference between France and the Netherlands is larger.

Within-sample forecasts do not indicate a best model. The results depend on the country, the sex, the main group and the length of the future period chosen.

4. Conclusion

The forecasts for France, using the Li-Lee and CFD method, show a life expectancy at birth between 86.3 and 89.1 years in 2050. For the Netherlands, the life expectancy at birth is between 86.3 and 87.9 years. The range of the life expectancy at birth in 2050 of the CFD forecasts is much smaller than the range of the Li-Lee forecasts (sexes combined). However, this does not hold for the forecasts for men.

The choice of the main group has a clear effect on the forecasted mortality, and seems to be larger than the choice of the method. However, the effect does not seem to depend on the amount of linearity in the observed data.

References

Booth, H & L. Tickle (2008): Mortality modelling and forecasting: a review of methods. Annals of Actuarial Science 3(1&2), 3–43.

Hyndman, R. J. (2010). Demography: Forecasting mortality, fertility, migration and population data. R package version 1.07. With contributions from Heather Booth and Leonie Tickle and John Maindonald. Retrieved from http://robjhyndman.com/software/demography

Hyndman, R.J., H. Booth & F. Yasmeen (2013): Coherent Mortality Forecasting: The Product-Ratio Method With Functional Time Series Models. Demography 50, 261–283.

Hyndman, R. J., & Ullah, M. S. (2007): Robust forecasting of mortality and fertility rates: A functional data approach. Computational Statistics & Data Analysis 51, 4942–4956.

Janssen, F., J.P. Mackenbach, & A.E. Kunst (2004): Trends in old-age mortality in seven European countries, 1950-1999. J Clin Epidemiol 57(2), 203–216.

Lee, R. D. & L. R. Carter (1992): Modelling and forecasting US mortality. Journal of the American Statistical Association 87(419), 659–671.

Li, N. & R. Lee (2005): Coherent mortality forecasts for a group of populations: an extension of the Lee-Carter method. Demography, 42(3), 575–94.

Pollard, J.H. (1987): Projection of age-specific mortality rates, Population Bulletin of the United Nations 21-22, 55-69.

Stoeldrajer, L., C. van Duin, L. van Wissen & F. Janssen (2013): Impact of different mortality forecasting methods and explicit assumptions on projected future life expectancy: The case of the Netherlands. Demographic Research, 29(13), 323-354.

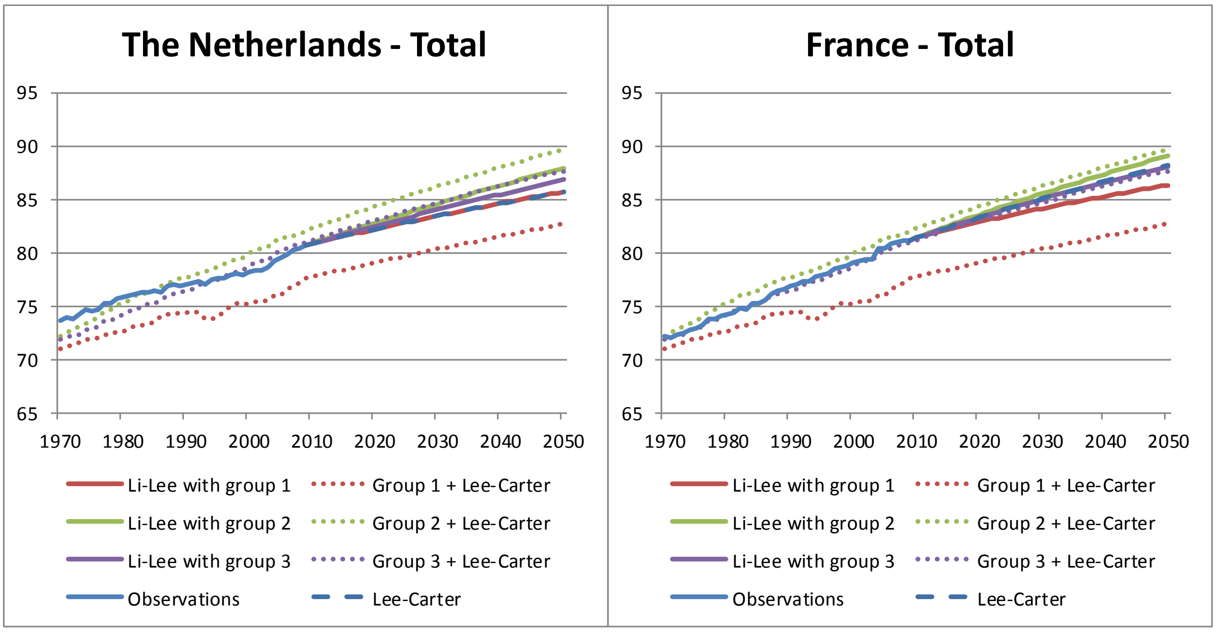
Tabeau, E. (2001): A review of demographic forecasting models for mortality. In E. Tabeau, A. Van Den Berg Jeths & C. Heathcote (Eds.), Forecasting mortality in developed countries: insights from a statistical, demographic and epidemiological perspective (pp. 1-32). Kluwer Academic Publishers, Dordrecht.

Wong-Fupuy, C. & S. Haberman (2004). Projecting Mortality Trends: Recent Developments in the United Kingdom and the United States. North American Actuarial Journal 8(1), 56–83.

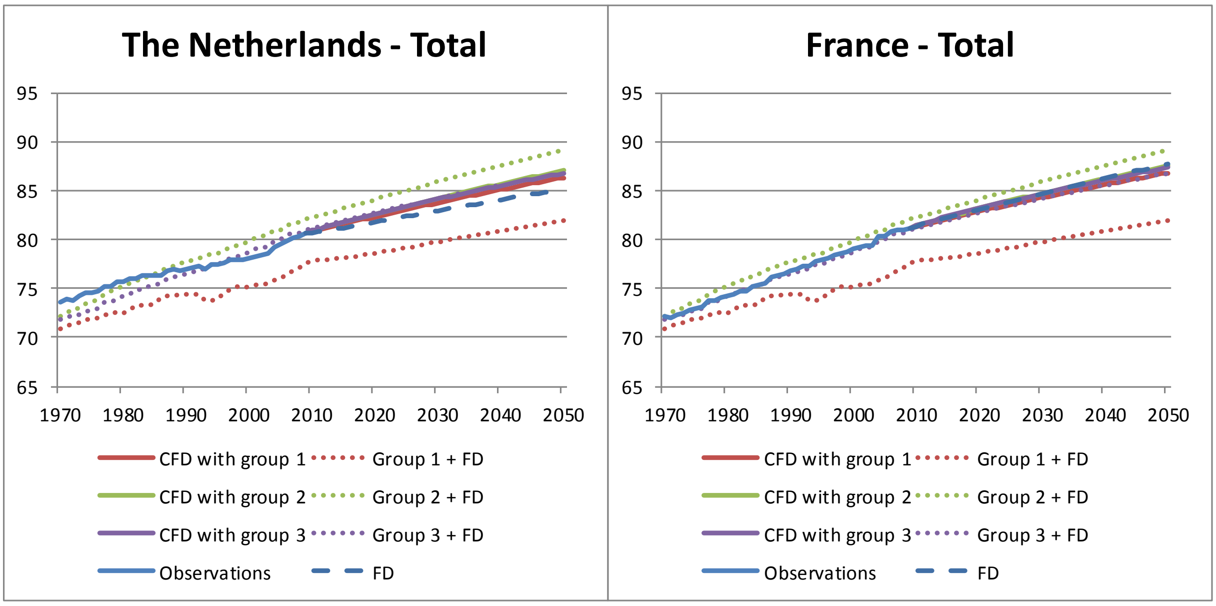
Figures

Figure 1 Life expectancy at birth, observations (1970-2009) and the two coherent forecasts compared (2010-2050), men and women together, the Netherlands and France

a. Lee-Carter method (individual) and Li-Lee method (coherent)



b. FD method (individual) and CFD method (coherent)



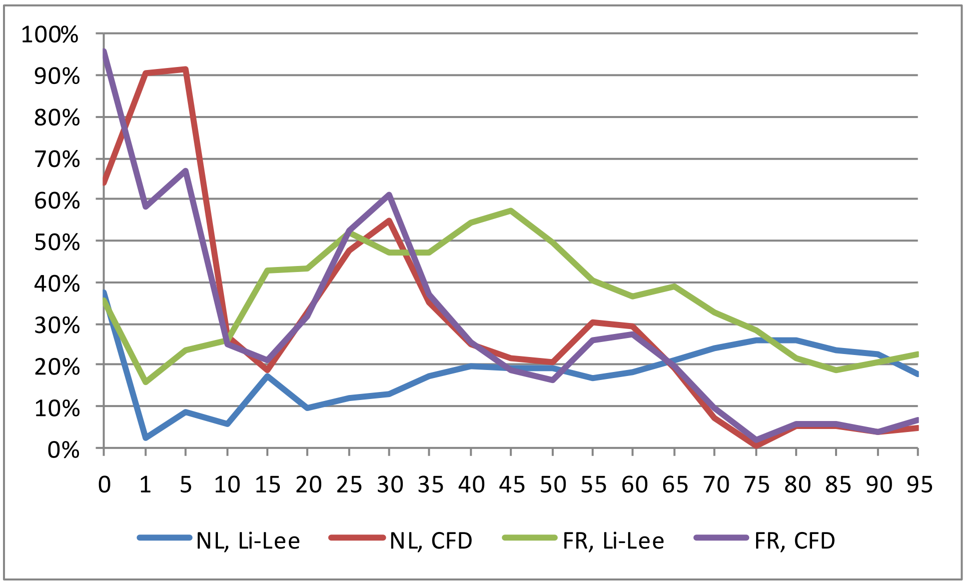
Group 1 = all 30 countries; Group 2 = 10 countries with highest life expectancy at birth in 2009; Group 3 = 14 western European countries

Table 1 Life expectancy at birth in 2050 using individual and coherent forecast methods, women, men, and men and women together, the Netherlands and France

			Women	Men	Total
The Netherlands	**Lee-Carter**		86.91	83.61	85.68
	Li-Lee	**Group 1**	87.04	83.68	85.70
		Group 2	89.58	85.39	87.93
		Group 3	88.40	84.87	86.81
		Mean (range)	88.34 (2.54)	84.65 (1.71)	86.81 (2.23)
	FD		86.38	86.04	85.11
	CFD	**Group 1**	87.17	84.20	86.35
		Group 2	89.05	85.64	87.02
		Group 3	88.45	86.28	86.78
		Mean (range)	88.22 (1.88)	85.37 (2.08)	86.72 (0.67)
France	**Lee-Carter**		90.89	85.04	88.19
	Li-Lee	**Group 1**	89.36	83.44	86.32
		Group 2	91.75	85.50	89.06
		Group 3	90.48	85.12	88.05
		Mean (range)	90.53 (2.39)	84.69 (2.06)	87.81 (2.74)
	FD		90.69	84.47	87.72
	CFD	**Group 1**	88.20	83.93	86.81
		Group 2	89.93	85.42	87.54
		Group 3	89.08	86.28	87.34
		Mean (range)	89.07 (1.73)	85.21 (2.35)	87.23 (0.73)

Group 1 = all 30 countries; Group 2 = 10 countries with highest life expectancy at birth in 2009; Group 3 = 14 western European countries

Figure 2 Range in the death rate in 2050 by age (divided by the non-coherent forecast at each age group)



Appendix

Table 1 Life expectancy at birth in 2009

Country	Life expectancy at birth in 2009		
	Women	Men	Total
Australia	84.17	79.73	81.96
Austria	82.84	77.44	80.22
Belarus	76.34	64.63	70.42
Belgium	82.41	77.15	79.83
Bulgaria	77.23	70.09	73.59
Canada	83.40	79.02	81.26
Czech Republic	80.26	74.15	77.24
Denmark	81.00	76.83	78.94
England & Wales	82.39	78.28	80.39
Estonia	80.05	69.74	75.11
Finland	83.12	76.51	79.85
France	84.45	77.79	81.19
Hungary	78.20	70.18	74.26
Ireland	82.22	77.26	79.74
Italy	84.21	79.21	81.81
Japan	86.49	79.62	83.13
Latvia	77.53	67.47	72.66
Lithuania	78.51	67.11	72.84
Netherlands	82.63	78.55	80.67
Norway	83.03	78.62	80.86
Poland	79.90	71.48	75.70
Portugal	82.45	76.41	79.49
Russia	74.65	62.68	68.61
Slovakia	78.95	71.36	75.20
Spain	84.56	78.49	81.55
Sweden	83.28	79.31	81.33
Switzerland	84.19	79.64	82.00
Taiwan	82.08	75.96	78.86
U.S.A.	81.03	76.14	78.63
Ukraine	74.79	64.32	69.56