

# Dynamic Forecast of Irregular Mortality Developments within a Bayesian Framework

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## Abstract

Forecasting irregular mortality developments is challenging, especially when long-time mortality trends change in the forecast years. Many countries in Central and Eastern Europe experienced an unsteady mortality development during the last 50 years, often including periods of stagnating and even decreasing life expectancy at birth; for instance, male life expectancy in Hungary decreased between the mid 1960s and the early 1990s, and strongly increased thereafter. As many mortality forecasting approaches extrapolate past trends, they fail to predict such trend changes. We try to overcome these problems with our novel mortality forecasting approach, which combines objective and subjective information in a Bayesian framework, i. e. we (1) use rates of mortality improvement (instead of death rates) to capture dynamic mortality developments, and we (2) can optionally complement a mortality trend in a country of interest with those of selected reference countries. These methodological refinements enable us to (1) incorporate flexible mortality dynamics and to (2) supplement and/or adjust them with expert judgment. In addition to a prospective forecast until 2050, we demonstrate in a retrospective application for Hungary that our model would have estimated Hungarian life expectancy more accurately than the original Lee-Carter model and two of its refinements proposed by Renshaw and Haberman: While the other applied models underestimate the progress in Hungarian life expectancy in 2009, after only 20 forecast years, by 4 to 5 years for women and by 6 to 8 years for men, we reduce this forecast error with our model to 2 years (for women and men) by only using the rates of mortality improvement; we then further improve the forecasting performance of our model by complementing the Hungarian mortality trend with that of West Germany, so that our forecasts exceed Hungarian life expectancy in 2009 by only 1 year for both sexes.

## **Introduction: Mortality trends in general and the case of Hungary**

In many highly developed countries, life expectancy at birth increases almost linearly for more than 150 years; Oeppen and Vaupel (2002) demonstrate that record life expectancy, i. e. the highest globally observed life expectancy at birth in a given year, increases for women about 2.5 years per decade. Vallin and Meslé (2009) argue that life expectancy rather follows a segmented trend line with different slopes due to different driving factors. However, these gains in life expectancy are due to mortality reductions at different ages; over the course of the twentieth century, relatively large survival improvements shifted from younger to higher ages: In the beginning of the twentieth century, mortality reductions were prevalent at younger ages, while from 1950 onwards, mortality reductions were (and are) mostly prevalent at advanced ages (Kannisto et al., 1994; Rau et al., 2008). For instance, in Japan, the current record holder, 80% of the increase in life expectancy at birth between 1990 and 2007 are due to mortality reductions at ages above 65, and even half of it are due to mortality reductions at ages above 80 (Christensen et al., 2009).

Many countries in Central and Eastern Europe experienced a different mortality development. From the 1960s onwards, life expectancy at birth often stagnated or even decreased, in particular for men. Grigoriev et al. (2013) find three main factors (in the literature) that drive this mortality development, namely socio-economic crisis, alcohol consumption and deterioration of the health-care system.

Figure 1 depicts life expectancy at birth for women (solid lines) and men (dashed lines) in Japan (black), Italy (blue), Sweden (yellow), West Germany (green) and Hungary (red) between 1950 and 2009. Japan is the current record holder with a life expectancy of 86.4 years for women and 79.33 years for men in 2009; Italian, Swedish and German life expectancies do not lag more than 4 years behind, whereas Hungarian life expectancy lags 8 years behind for women and 9 years for men. Although each of these countries experienced an increase in life expectancy between 1950 and 2009, they differ in their slope. Except Hungary, all countries presented here experienced a continuous increase in life expectancy, but this increase was much more pronounced in Japan, Italy and Germany than in Sweden. In contrast, Hungary experienced an unsteady mortality development: Periods of increasing life expectancy were followed by periods of stagnating and even decreasing life expectancy.

Such a mortality development—as observed in Hungary—might give rise to speculation on bad data quality. But neither the documentation of Hungarian mortality data in the Human Mortality Database by Radnóti and Jasilionis (2011), nor the study on the quality of mortality data at advanced ages in countries of the Kannisto-Thatcher database by Jdanov et al. (2008) support this argument. In contrast, Jdanov et al. (2008) assign Hungary to the group of countries with acceptable mortality data quality—that is the second best group (out of four groups) that also includes countries like Austria and England & Wales.

According to the World Health Organization (2010), behavioral factors such as smoking (see also Preston et al. (2010)), nutrition and exercise are major health challenges for Hungary as reflected in the death rates among men aged 40 to 65 years. The downward trend of

the contour lines in the upper right panel of Figure 2 illustrates that death rates have increased at ages 40 to 65 during the last three decades of the twentieth century. Despite the reversal of this negative trend in the most recent 15 years, one observes the same death rates now for 50 year-olds as already 50 years ago. An even better description of this mortality development is given in the bottom panels of Figure 2 by the rates of mortality improvement, which we define as the time-derivative of the age-specific death rates. The grey and black colors indicate the survival worsening for both sexes in the medium adult ages between the mid 1960s and the early 1990s, whereas the yellow, orange and red colors indicate the changing mortality trend thereafter, i. e. the strong survival improvements from the early 1990s onwards.

### **Forecasting Mortality**

Forecasting mortality is difficult per se, but irregular developments, as the ones observed in Hungary, are even more challenging.

**Regular mortality development** Although regular increases in life expectancy at birth seem to be easy to forecast, many approaches fail to capture underlying mortality developments like a dynamic age shift in mortality reductions, i. e. when older ages increasingly experience survival improvements over time. For instance, the widely adopted model by Lee and Carter (1992) uses a time-invariant age schedule, which determines relative progress in mortality between ages for all forecast years; that is why it cannot mirror dynamic age shifts that are prevalent in many highly developed countries. This shortcoming also applies to various extensions of the Lee-Carter model that have been proposed by, e. g., Renshaw and Haberman (2003; 2006). Recent approaches try to overcome this inflexibility with various innovations. For instance, Mitchell et al. (2013) and Haberman and Renshaw (2012) use the predictor structure of the original Lee-Carter model to forecast the rates of mortality improvement instead of the death rates to catch dynamic mortality developments.

**Irregular mortality development** Irregular developments in life expectancy at birth are even more difficult to forecast. Many forecasting approaches fail to predict such unsteady mortality developments, in particular, when mortality tends to change its long-time trend in the forecast years. Recent approaches try to circumvent such problems by jointly forecasting mortality of multiple populations (or countries) like Li and Lee (2005) or Cairns et al. (2011).

We propose a novel mortality forecasting approach in a Bayesian framework, which combines these recently developed ideas, i. e. (1) to catch dynamic mortality developments, we use the rates of mortality improvement instead of the death rates to forecast mortality and (2) to forecast unsteady mortality developments, we optionally complement the mortality trend in a country of interest with those of selected reference countries.

To test the forecasting performance of our Bayesian model under irregular mortality conditions, we generate in-sample mortality forecasts for Hungary: We take Hungarian mortality

data by single age and sex from 1970 to 1990 in order to forecast them from 1991 to 2009.<sup>1</sup> We then use forecast errors, i. e. the difference between the forecasted and the actually observed life expectancies at birth, to compare the forecasting performance of our model with that of the original Lee-Carter model and with three of its extensions (h0, h1 and h2) proposed by Renshaw and Haberman (2003; 2006).

To generate these in-sample mortality forecasts with the Lee-Carter model and its extensions by Renshaw and Haberman, we use the freely available implementation by Timothy Miller (<http://www.demog.berkeley.edu/~tmiller/research/forecasts/mort.forecast.module.s>) and the *ilc* R-package by Butt and Haberman (2010).

Figure 3 depicts observed (black) Hungarian life expectancy at birth from 1950 to 2009 as well as the in-sample forecasts of our model (blue), of the Lee-Carter model (magenta) and of its three extensions by Renshaw and Haberman (green, yellow and red). The corresponding forecast errors from 1991 to 2009 are shown in Figure 4. These two figures illustrate that our model estimates the observed life expectancy in Hungary more accurately than the other models. The latter systematically underestimate the progress in life expectancy for women and, even more pronounced, for men from the early 1990s onwards. For instance, while the other models underestimate Hungarian life expectancy in 2009 by 4 to 5 years for women and by 6 to 8 years for men, our model underestimates it by only 2 years for both sexes, if we only use the rates of mortality improvement (dashed blue lines). We can even further improve the forecasting performance of our model by complementing the Hungarian mortality trend with that of West Germany so that our model exceeds Hungarian life expectancy in 2009 by only 1 year for both sexes (solid blue lines).<sup>2</sup>

We also generate out-of-sample forecasts for Hungarian mortality, taking mortality data from 1970 to 2009 in order to forecast them from 2010 to 2050 with our model as well as with the Lee-Carter model and with its three variants by Renshaw and Haberman. Figure 5 depicts the results (with the same color coding scheme as in the two previous figures): All models forecast a further increase in life expectancy at birth for both sexes, but with a different slope. Our model forecasts life expectancy to increase by 10 years for women and by 12 years for men between 2010 and 2050, whereas the other models forecast a much slower increase for life expectancy with 4.5 years for women and with 3 years for men in the same time.

## Acknowledgement

The European Research Council has provided financial support under the European Community's Seventh Framework Programme (FP7/2007-2013) / ERC grant agreement no. 263744.

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<sup>1</sup>Such detailed Hungarian mortality data are provided by the Human Mortality Database (2013).

<sup>2</sup>We also used the model by Li and Lee (2005) to generate in-sample mortality forecasts for Hungary (in coherence with the mortality trend of West Germany), but we do not illustrate these results, because (1) male forecasts did not run (probably due to the strong downward trend in the base period) and (2) female forecasts seem to be implausible as they predict that female life expectancy at birth is likely to increase only marginally from 76.42 years in 1991 to 76.52 years in 2009.

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### Life expectancy at birth

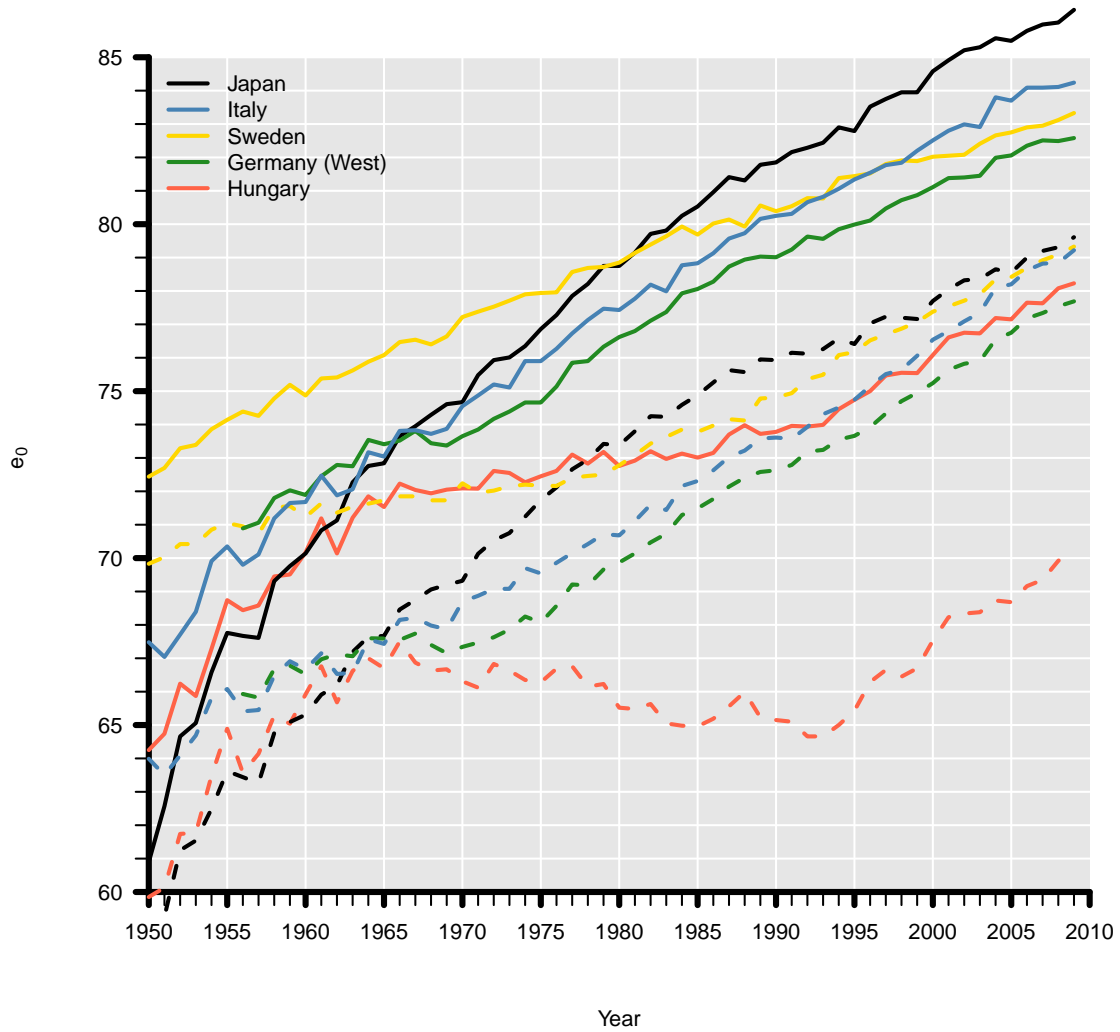


Figure 1: Observed life expectancy at birth for women (solid lines) and men (dashed lines) in Japan (black), Italy (blue), Sweden (yellow), West Germany (green) and Hungary (red) from 1950 to 2009. Data source: Human Mortality Database (2013).

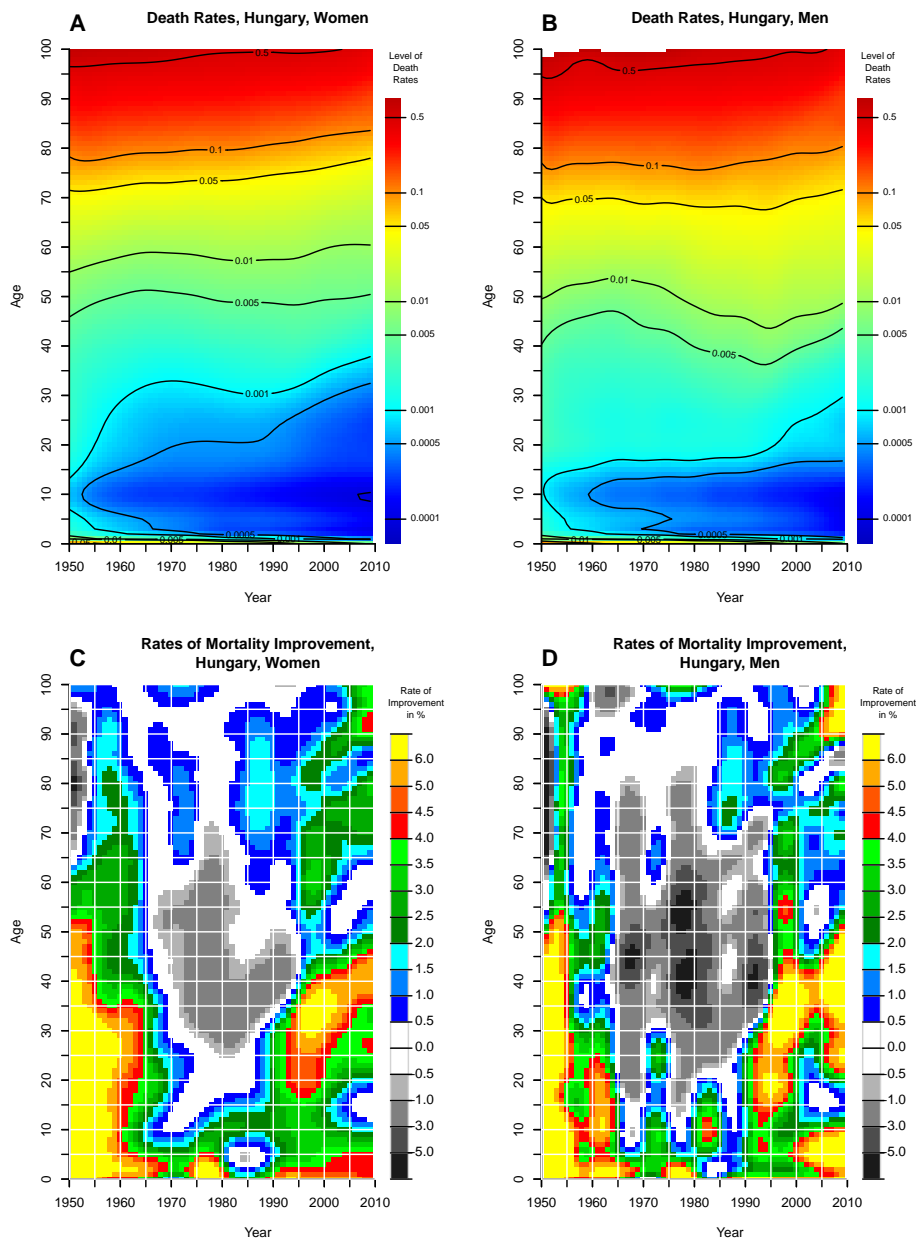


Figure 2: Upper Panel: Hungarian death rates for women (A) and men (B). Lower Panel: Rates of mortality improvement in Hungary for women (C) and men (D). Source: Own estimation based on data from the Human Mortality Database (2013).



### Life expectancy at birth, Hungary

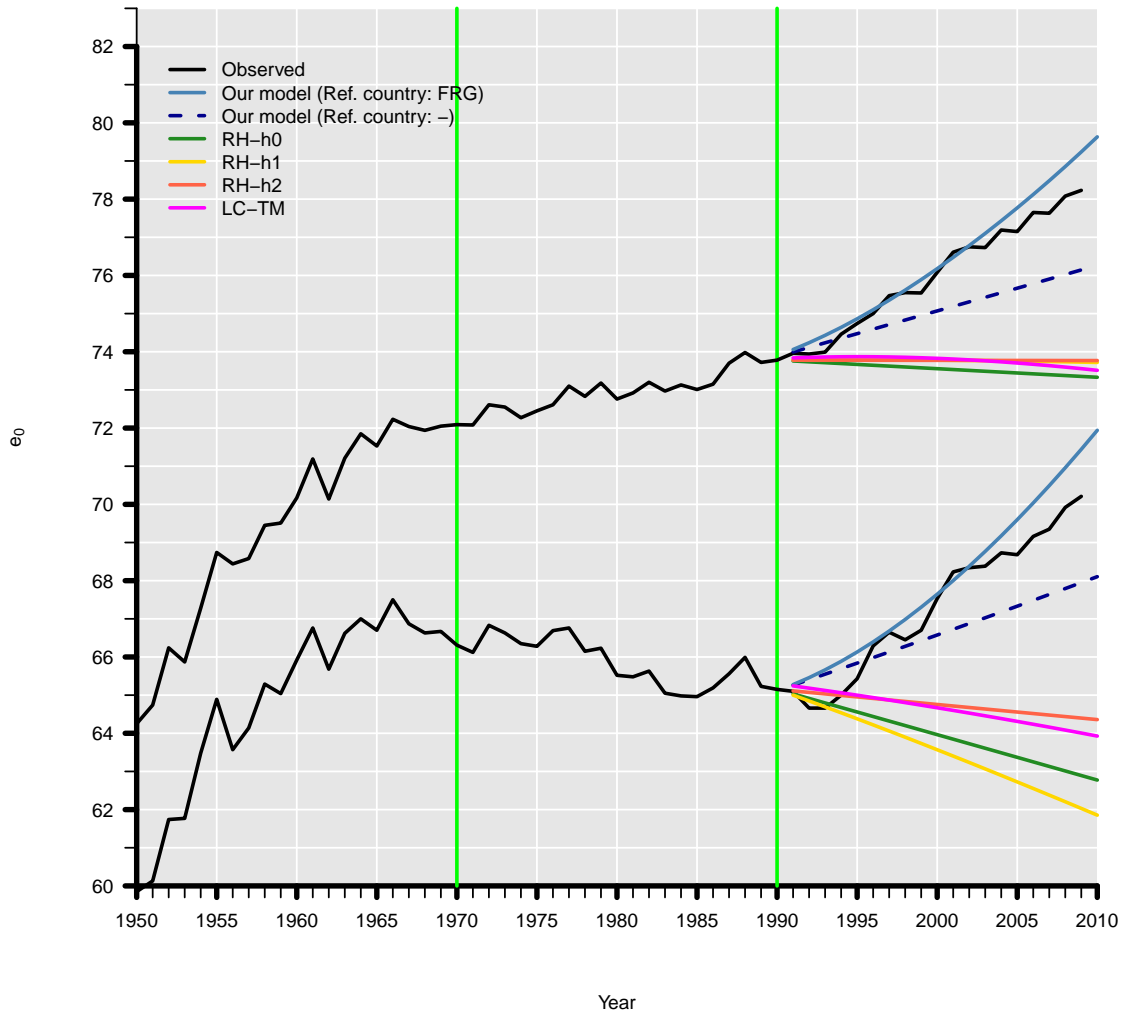


Figure 3: Observed (black) and forecasted life expectancy at birth ( $e_0$ ) of our model (blue), of the original Lee-Carter model (magenta) and of its variants h0 (green), h1 (yellow) and h2 (red) proposed by Renshaw and Haberman for women (upper graph) and men (lower graph). For this in-sample forecast, we take mortality data from 1970 to 1990 (green vertical lines) in order to forecast them from 1991 to 2009. In our model, we do not (dashed blue line) and we do complement the mortality trend for Hungarian women and men with those of West German women and men (solid blue line).

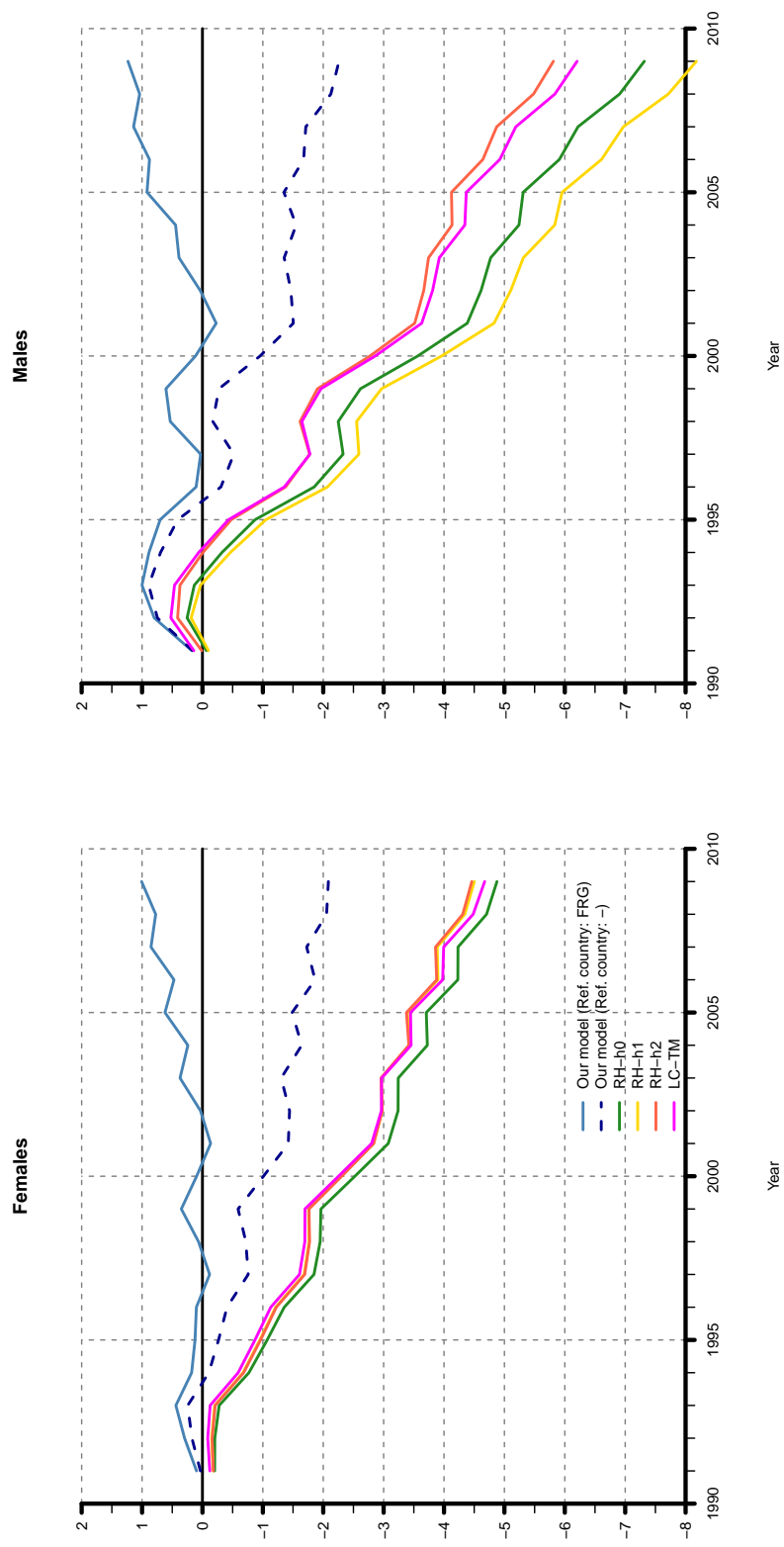


Figure 4: Forecast errors, i. e. differences between forecasted and observed life expectancies at birth, for Hungarian women (right) and men (left) from 1991 to 2009 according to our model—with the reference country West Germany (solid blue) and without a reference country (dashed blue)—, to the Lee-Carter model (magenta) and according to its three extensions, h0 (green), h1 (yellow) and h2 (red), proposed by Renshaw and Haberman.

### Life expectancy at birth, Hungary

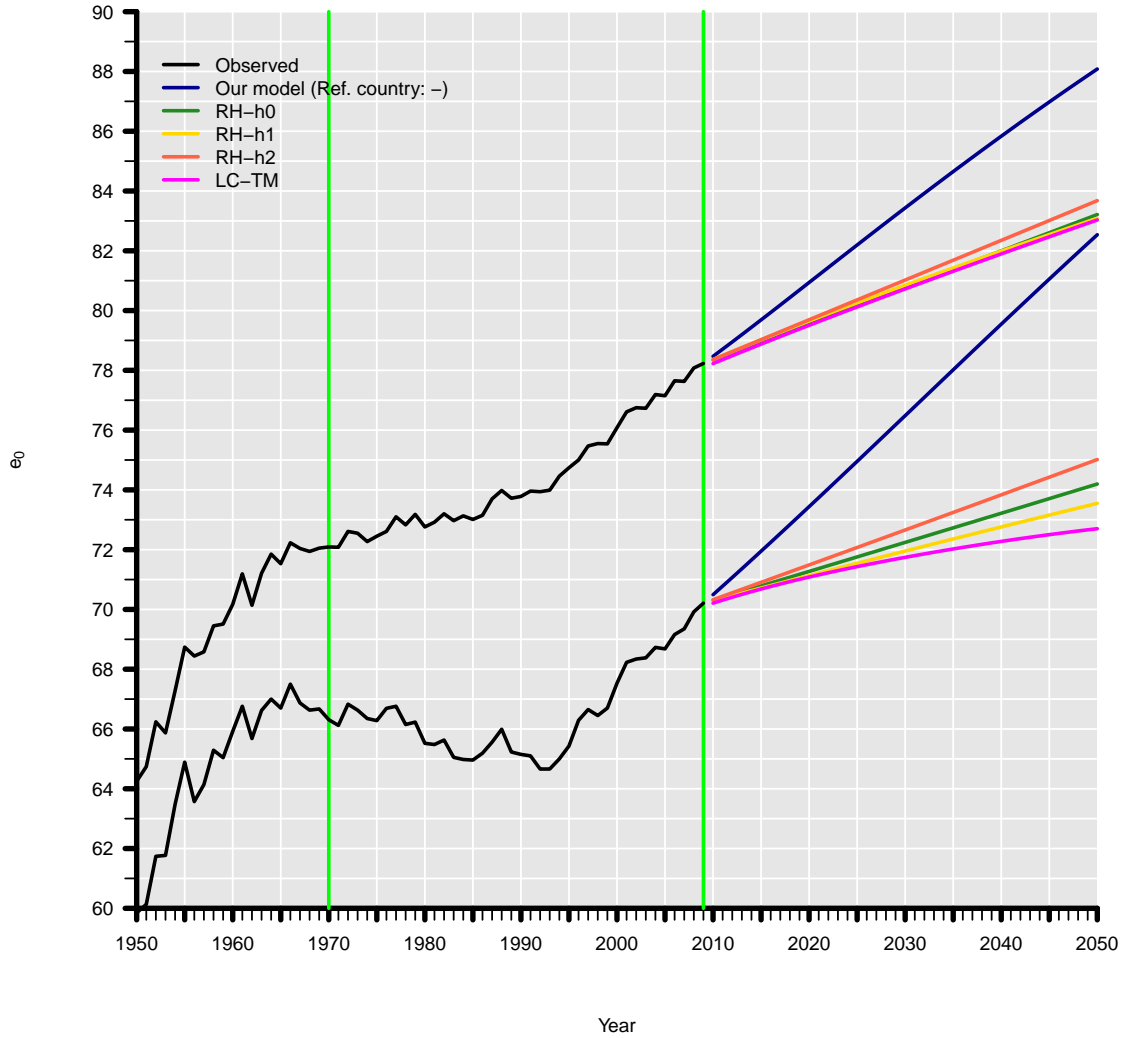


Figure 5: Observed (black) and forecasted life expectancy at birth ( $e_0$ ) of our model (blue), of the original Lee-Carter model (magenta), of its coherent variant (orange) proposed by Li and Lee and of its variants h0 (green), h1 (yellow) and h2 (red) proposed by Renshaw and Haberman for women (upper graph) and men (lower graph). For this out-of-sample forecast, we take mortality data from 1970 to 2009 (green vertical lines) in order to forecast them from 2010 to 2050. In our model and in the coherent forecast, we complement the mortality trend for Hungarian women and men with those of West German women and men.