Pace and shape decomposition of mortality

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Abstract

We study two dimensions of mortality: pace and shape. These two components of mortality change inform us about the timing and age patterns of mortality respectively. The aim of this study is to decompose changes in life expectancy into pace and shape effects. Our new approach allows us to differentiate between the two underlying processes in mortality and their relevance to understand the dynamics of mortality.

1 Background

In order to explain the dynamic behind changes in mortality, demographers have developed several techniques to decompose changes in life expectancy by different components of mortality. Some methods focus on discrete differences between two life tables (Arriaga, 1984; Pollard, 1982; Pressat, 1985) while others considered continuous changes (Keyfitz, 1977; Vaupel and Canudas-Romo, 2003; Vaupel, 1986). All those methods focus on decomposing mortality changes by age or cause of death.

A recent study by Baudisch (2011) emphasizes the importance to distinguish between two dimension of aging for inter-species comparison: pace and shape. Pace refers to the time aspect of aging; "it is the time-scale on which mortality progresses" (Baudisch, 2011, p.1). The shape refers to the age-pattern of mortality or how mortality changes with age. In human demography, this framework could be related with concepts introduce by the shifting mortality and compression of mortality hypotheses. The shifting mortality hypothesis suggests a delay in the mortality schedule, but with a shape which remains the same (Bongaarts and Feeney, 2002, 2003; Canudas-Romo, 2008). On the other hand, the compression of mortality hypothesis suggests a change in variability in the age at death, manifest by a rectangularization of the survival curve shape and deaths occurring in a shorter age-interval (Fries, 1980; Kannisto, 2000).

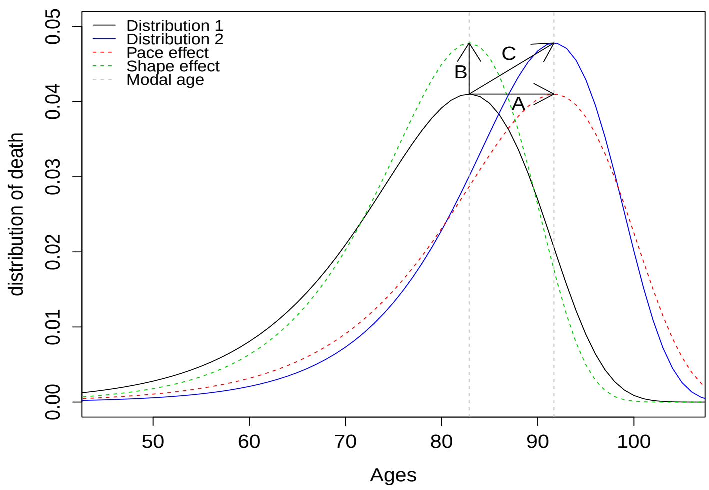
Changes in mortality can then be produced by a change in pace or by a change in shape and more commonly by changes in both dimensions simultaneously. Those two dimension have the potential to inform about different mortality dynamics: shift in mortality schedule and changes in variability. This research aims to study the respective impact of changes in pace and shape on life expectancy. We introduce a new methodology to decompose the change in life expectancy between two distributions by a pace and a shape contribution.

2 Methods

2.1 Definitions and concept

A change in pace is defined here as a change in the modal age at death, while a change in shape refers to a change in the slope of the hazard function. Changing the slope of the hazard distribution also changes the shape of the density and survival distributions. If there is a change of mortality between two distributions (in Figure 1 as C), we define the "pace effect" as the hypothetical change in mortality produced if only the modal age at death would have changed between those two distributions (in Figure 1 as A). The "shape effect" refers to the hypothetical change in mortality produced if only the slope of the hazard function would have changed from one distribution to another (in Figure 1 as B). These concepts are illustrated in Figure 1 presenting density functions of the distribution of deaths for a simulated Gompertz mortality.

Figure 1: *Illustration of the pace and shape effects in the density function of the distribution of deaths for simulated data from a Gompertz model with intercepts* $\alpha_1 = 0.000097$ *and* $\alpha_2 = 0.000007$, *slopes* $\beta_1 = 0.1115$ *and* $\beta_2 = 0.1300$ *and modal ages at death* $M^1 = 82.9$ *and* $M^2 = 91.9$



We present first the methodology based on Gompertz model. We need to start from two known hazard functions (μ_x) defined as:

$$\mu_x^1 = \alpha^1 e^{\beta^1 x}$$

$$\mu_x^2 = \alpha^2 e^{\beta^2 x}$$

where α^i and β^i correspond to the intercept and slopes in the Gompertz model for distribution i. If mortality can be decomposed by those two dimensions, then we can find the equivalences:

$$e_x^2 - e_x^1 = (e_x^s - e_x^1) + (e_x^p - e_x^1) = \Delta^s + \Delta^p \tag{1}$$

and

$$\mu_x^2 - \mu_x^1 = (\mu_x^s - \mu_x^1) - (\mu_x^p - \mu_x^1) \tag{2}$$

where e_x^i is the life expectancy at age x for distribution i, and we denote the life expectancy from the pace and shape effects as e_x^p and e_x^s respectively.

2.2 Shape effect

To estimate the shape effect we assume that the modal age at death (M) stays constant at the value of M^1 during the change between the two distributions. However, the shape changes from a slope of mortality of β^1 to β^2. The modal age at death in a Gompertz distribution is defined as:

$$M = \frac{1}{\beta} ln(\frac{1}{\alpha}).$$

Lets M^s and β^s be the modal age at death and the slope when only shape changes are observed. Given the constraints of fixed mode, $M^s = M^1$, and changing slope, $\beta^s = \beta^2$, the parameter α^s is restricted to be:

$$\alpha^s = \frac{1}{e^{\frac{\beta^2}{\beta^1} ln(\frac{1}{\alpha^1})}}.$$

Resulting in a shape hazard effect, μ_x^s:

$$\mu_x^s = e^{\beta^2 (x - M^1)}. \tag{3}$$

It can be mathematically shown that equation 3 equals to:

$$\mu_x^s = \mu_x^2 e^{\beta^2 \Delta}, \tag{4}$$

in other words a shift of μ_x^2 by Δ, where $\Delta = M^2 - M^1$.

Under a Gompertz distribution, the survival function (l_x) and life expectancy at age x (e_x) are:

$$l_x = e^{-\int_x^\omega \mu_a da} = e^{-\frac{\alpha}{\beta}[e^{\beta x} - 1]} \tag{5}$$

$$e_x = \int_x^\omega l_a da. \tag{6}$$

Life expectancy where only shape of mortality is operating is calculated from substituting equation 4 in the equation 6 for life expectancy as

$$e_x^s = \int_x^\omega e^{\frac{-\alpha^2}{\beta^2}[e^{\beta^2 (a + \Delta)} - 1]} da. \tag{7}$$

The shape contribution to the change in life expectancy can be calculated as $(e_x^s - e_x^1) = \Delta_x^s$.

2.3 Pace effect

The pace effect is estimated in a similar fashion as the shape effect. However, now we assume a change in the modal age at death from M^1 to M^2, but the slope of mortality remains fixed to β^1: $M^p = M^2$ and $\beta^p = \beta^1$. The hazard for the pace effect is:

$$\mu_x^p = e^{\beta^1(x-M^2)}, \tag{8}$$

or in terms of difference in modal ages at death as

$$\mu_x^p = \mu_x^1 e^{-\beta^1 \Delta}. \tag{9}$$

The pace contribution to the change in life expectancy would then be $(e_x^p - e_x^1) = \Delta_x^p$, where the expression for life expectancy when only the pace effect is operating (e_x^p) is found by substituting equation 9 in equation 6 as

$$e_x^p = \int_x^\omega e^{\frac{-\alpha^1}{\beta^1}[e^{\beta^1(a-\Delta)}-1]} da. \tag{10}$$

2.4 Interaction effect

To have the equivalence shown by equation 2, an interaction effect needs to be added. By solving equation 2, we find an interaction effect equal to :

$$e^{(\beta^1-\beta^2)(M^2-M^1)}. \tag{11}$$

In most cases, this number is going to be small and negligible as the differences in slopes are small-scaled and, in most of cases, negative.

3 Illustrations: effect on mortality of eliminating cancer for French women in 1999

3.1 Using Gompertz

Figure 2 shows women's mortality for France in 1999 with and without cancer, fitted with a Gompertz model. The respective life expectancy at age 15 for the four distributions (total ($e^t = e^1$), without cancer ($e^{-i} = e^2$), pace (e^p) and shape (e^s)) are : $e^t = 63.61$, $e^{-i} = 70.05$, $e^p = 65.75$ and $e^s = 67.91$. The interaction factor being very small, Δ^s and Δ^p sum up very closely to the difference between e^{-i} and e^t. Around 2/3 of the change of eliminating cancer results from a shape effect and 1/3 from a pace effect.

$$e^{-i} - e^t \approx \Delta^s + \Delta^p = (e^s - e^t) + (e^p - e^t)$$
$$6.45 \approx 4.30 + 2.14$$

3.2 Applied to observed data

The Gompertz model is not offering a good fit to the mortality without cancer in France when a bigger range of ages is considered than ages 30 to 90. It could however be possible to estimate the pace and shape contribution to the change in life expectancy from the observed μ_x and in a discrete way, by using the life table aging rates by age (LAR_x), which equal to $ln(\mu_x) - ln(\mu_{x-1})$ instead of the β from the Gompertz. The modal age at death is then calculated from the observed death distribution.

Figure 2: *Hazard, density and survival functions comparison between total mortality and mortality without cancer, fitted with Gompertz model, and their pace and shape effect, French women in 1999*

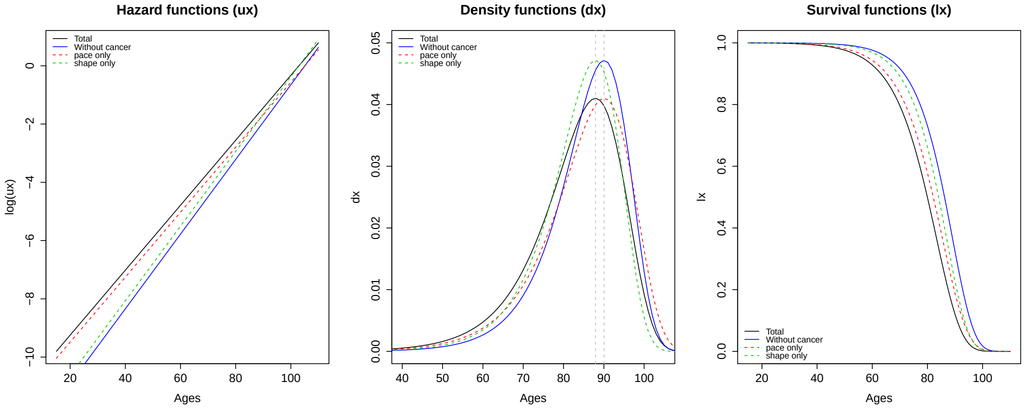
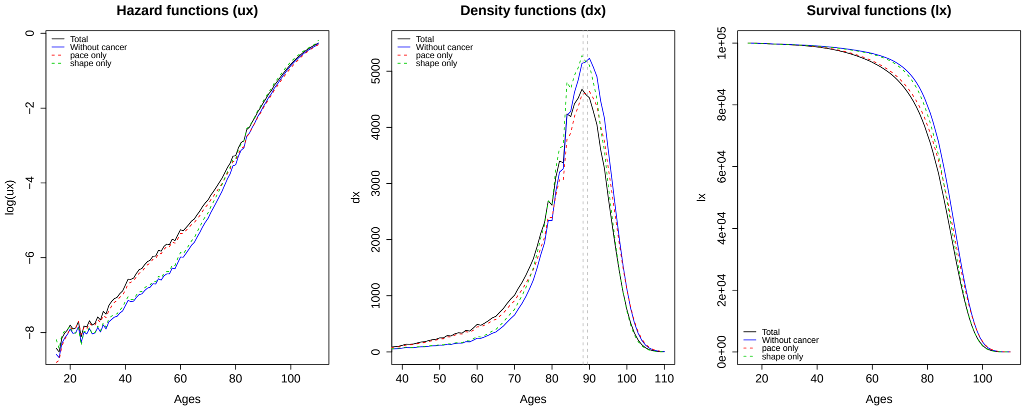


Figure 3: *Observed hazard, density and survival functions comparison between total mortality and mortality without cancer and their pace and shape effect, French women in 1999*



The observed life expectancy at age 15 is then : $e^t = 68.00$, $e^{-i} = 70.92$, $e^p = 69.14$ and $e^s = 69.77$. As with the Gompertz, changes in eliminating cancer are still driven by the changes in shape (3/5 of the change).

$$e^{-i} - e^t \approx \Delta^s + \Delta^p = (e^s - e^t) + (e^p - e^t)$$

$$2.92 \approx 1.77 + 1.15$$

In further research, the above methodology will also be applied to compare pace and shape changes through years, more populations, and other causes of death.

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